EJERCITACION

TRANSFORMADA DE LAPLACE

PARTE 1: TRANSFORMADA DIRECTA DE LAPLACE - PROPIEDADES

Ejercicio n° 1:

Calcule las Transformadas de Laplace de las siguientes funciones, utilizando la tabla y la propiedad de linealidad:

a) $y(t) = 3 + 5\,t - 2\,t^2 + t^3 \quad ; t>0$

b) $y(t) = \frac{1}{4}\,(e^{4t} - 1) \quad ; t>0$

c) $y(t) = \frac{1}{a^2}\,(e^{at} - at - 1) \quad ; t>0$

d) $y(t) = 4\cos(3t) - 5\,\text{sen}(2t) \; ; \; t>0$

Ejercicio n° 2:

Utilizando las propiedades de la Transformada de Laplace halle $\mathcal{L}[f(t)]$

a) $f(t) = t^2\,e^{4t} \; ; t>0$

b) $f(t) = 5\,e^{-5t}\cos(3t) \; ; t>0$

c) $f(t) = e^{-t}\,t\,\text{sen}(2t) \; ; t>0$

d) $f(t) = (t-1)^4 \; ; \; t>1$

e) $f(t) = e^{-2t}\,\text{sen}(2t) + t^2\,e^{3t} \; ; t>0$

f) $f(t) = 2\,t^2\,e^{-t} - t + \cos(4t)$

Ejercicio n° 3:

Calcule las Transformadas de Laplace aplicando adecuadamente alguna propiedad conveniente o algún paso algebraico:

a) $y(t) = \frac{1}{a^2} \; ; \; 0 < t < a$

b) $y(t) = \text{sen}\,(5t + \pi/3) \; ; t>0$

c) $y(t) = \cos^3(t) \quad ; t > 0$

d) $y(t) = a^t \quad ; t > 0 \; ; a \in R^+$

e) $y(t) = (t-1)^4 \quad ; t > 0$

f) $y(t) = \text{sen}(5t)\cos(2t) \; ; t > 0$

Ejercicio n° 4:

Sea $f(t) = k\,\text{sen}(2t+\frac{\pi}{6})$. Determine el valor de k sabiendo que $\lim\limits_{s\to\infty} s\,F(s) = 4$

Recuerde el Teorema de valor inicial.

PARTE 2: ANTITRANSFORMADA DE LAPLACE - PROPIEDADES

☑ Ejercicio n° 5:

Calcule las Antitransformadas de Laplace de las siguientes funciones:

a) $Y(s) = \dfrac{25}{s^3}$ b) $Y(s) = \dfrac{8}{s^2+3} + \dfrac{1}{s}$ c) $Y(s) = \dfrac{12}{s-3} - \dfrac{8s}{s^2+4}$ d) $Y(s) = \dfrac{-1}{s+3}$

e) $Y(s) = \dfrac{6}{(s-1)^4}$ f) $Y(s) = \dfrac{s+3}{s^2+6s+13}$ g) $Y(s) = \dfrac{s-1}{s^2+2s+2}$ h) $Y(s) = \dfrac{4s}{(s^2+4)^2}$

☑ Ejercicio n° 6:

Calcule las Antitransformadas de Laplace de las siguientes funciones previamente separando en fracciones simples:

a) $F(s) = \dfrac{s+7}{s^2-6s+5}$ b) $F(s) = \dfrac{s^2+2s+2}{s^2+3s+2}$ c) $F(s) = \dfrac{1}{(s+2)^2(s+1)}$

d) $F(s) = \dfrac{s^2+9s+19}{(s+1)(s+2)(s+4)}$ e) $F(s) = \dfrac{2\,e^{-0.5s}}{s^2+6s+3}$ f) $F(s) = \dfrac{6s^2-13s+12}{s(s-1)(s-6)}$

g) $s^2\,F(s) - 4\,F(s) = \dfrac{s}{s+1}$ h) $F(s) = \dfrac{7s^2-41s+84}{(s-1)(s^2-4s+13)}$

i) $F(s) = \dfrac{-2s^2+8s-14}{(s+1)(s^2-2s+5)}$ j) $F(s) = \dfrac{(4s+2)\,e^{-2s}}{(s-1)(s+2)}$

☑ Ejercicio n° 7:

a) Aplicando el Teorema de valor Final, halle el valor de f(t) para $t \to \infty$, siendo:

$F(s) = \dfrac{10}{s^2+s}$ Verifique el resultado obtenido.

b) Dada: $F(s) = \dfrac{1}{(s+2)^2}$ halle f(0) por Teorema del valor inicial. Verifique los resultados obtenidos.

c) Dada $F(s) = \dfrac{2s+1}{s^2+2s}$, halle el valor de f(t) para: i) t= 0 ii) t= ∞ iii) t= 0.5

Verifique i) y ii) con los Teoremas de Valor inicial y final.

PARTE 3: RESOLUCION DE ECUACIONES DIFERENCIALES

Ejercicio n° 8:

Resuelva las siguientes Ecuaciones Diferenciales por Transformada de Laplace:

a) $2\,x''(t) + 7\,x'(t) + 3\,x(t) = 6$ con $x(0)=0 \wedge x'(0)=0$

b) $x''(t) + 3\,x'(t) + 6\,x(t) = 2$ con $x(0)=0 \wedge x'(0)=0$

c) $3\,y'(t) + 5\,y(t) = 6$ con $y(0)=0$

d) $x'(t) + x(t) = e^{-2t}$ con $x(0)=0$

e) $y''(t) + 4\,y'(t) + 4\,y(t) = 3\,x'(t) + 2\,x(t)$ con $x(t) = e^{-5t} \wedge y(0)=0 \wedge y'(0)=0$

f) $y''(t) + 6\,y'(t) + 18\,y(t) = 13\,e^{-5t}$ con $y(0)=1 \wedge y'(0)=-2$

g) $y''(t) - 2\,y'(t) + 5\,y(t) = -8\,e^{-t}$ con $y(0)=2 \wedge y'(0)=12$

h) $y''(t) - 3\,y'(t) + 2\,y(t) = \cos(t)$ con $y(0)=0 \wedge y'(0)=-1$

i) $y''(t) + 9\,y(t) = \cos(2t)$ con $y(0)=1 \wedge y(\frac{\pi}{2})=-1$

OPTATIVO: $y''(t) + 3\,y'(t) + 2\,y(t) = t\,\mathrm{sen}(2t)$ con $y(0)=0 \wedge y'(0)=0$

Ejercicio n° 9:

Resuelva los siguientes sistemas de Ecuaciones Diferenciales por medio de la Transformada de Laplace:

a) $\begin{cases} y'(t) - \frac{1}{2}\,x(t) = 1 \\ x'(t) + 2\,y(t) = 2t \end{cases}$ con $x(0) = 0,\ y(0) = 0$

b) $\begin{cases} x'(t) = x(t) - y(t) \\ y'(t) = 2\,x(t) + 4\,y(t) \end{cases}$ con $x(0) = -1,\ y(0) = 0$

c) $\begin{cases} x'(t) + 2\,y(t) = 0 \\ x'(t) - y'(t) = 0 \end{cases}$ con $x(0) = -1,\ y(0) = 2$

d) $\begin{cases} x'(t)+2y'(t)+5x(t)=0 \\ y'(t)+y(t)+2x(t)=0 \end{cases}$; $y(0) = 1 \wedge x(0) = 0$

e) $\begin{cases} 2\,x'(t) - y'(t) = 4\,t - 3 \\ x'(t) = y(t) - t \end{cases}$; $y(0)=2 \wedge x(0)=1$

OPTATIVO:

$\begin{cases} y'(t) + 2\,z'(t) = t \\ y''(t) - z(t) = e^{-t} \end{cases}$ con $y(0) = 3,\ y'(0) = -2,\ z(0) = 0$

PARTE 4: CONVOLUCION

Ejercicio n° 10:

Halle $\mathcal{L}\left[\int_0^t u\cos(u)\, e^{(t-u)}\, du\right]$ utilizando el Teorema de Convolución.

Ejercicio n° 11:

Antitransforme utilizando Convolución:

a) $Y(s) = \dfrac{6}{(s+1)(s^2+4)}$ b) $Y(s) = \dfrac{1}{(s^2+1)^2}$ c) $Y(s) = \dfrac{s^2}{(s^2+1)^2}$

Ejercicio n° 12:

Resuelva las siguientes ecuaciones integrales e integrodiferenciales por medio de la Transformada de Laplace, utilizando el Teorema de Convolución:

a) $y(t) + 4\int_0^t (t-u)^2\, y(u)\, du = t^2$ b) $y(t) = t + 2\int_0^t \cos(t-u)\, y(u)\, du$

c) $\int_0^t y(u)\,(t-u)\, du = \dfrac{t}{2} - \dfrac{\operatorname{sen}(2t)}{4}$ d) $y(t) = \int_0^t y(u)\, du + \cos(t)$

e) $\int_0^t y(u)\,(t-u)\, du + y'(t) = \dfrac{t^4}{12} + t + 1$ con $y(0) = -1$

f) $y'(t) = 1 - \int_0^t y(u)\, e^{2(t-u)}\, du$ con $y(0) = 1$

PARTE 5: EVALUACION DE INTEGRALES

Ejercicio n° 13:

Calcule el valor de las siguientes integrales utilizando Transformada de Laplace:

a) $\int_0^\infty t^3 e^{-t} \operatorname{sen}(t)\, dt$ b) $\int_0^\infty e^{-3t}\, t \operatorname{sen}(t)\, dt$ c) $\int_0^\infty t^2 e^{-2t}\, dt$

d) $\int_0^\infty \dfrac{e^{-3t} - e^{-6t}}{t}\, dt$ e) $\int_0^\infty t\, e^{-2t} \cos(4t)\, dt$

EJERCICIOS PARA MARCAR LA RESPUESTA CORRECTA, tomados en Finales:

☑ Ejercicio n° 14:

Marcar la única respuesta correcta de cada ítem:

1.	Sabiendo que $g(t) = e^{5t} f(t)$ entonces: a) $\lim_{s\to\infty} s\,F(s) = \lim_{s\to\infty} s\,G(s)$ b) $\mathcal{L}[f(t)] = G(s-5)$ c) $\mathcal{L}[g(t)] = F(s) \bullet \frac{1}{s-5}$ d) Ninguna es correcta
2.	Sabiendo que $F(s) = \mathcal{L}[f(t)] \wedge G(s) = \mathcal{L}[g(t)]$ entonces: a) $\mathcal{L}[f(t) \bullet g(t)] = F(s) \bullet G(s)$ b) Si $f(t) = \delta(t)$ entonces $F(s) \bullet G(s) = G(s)$ c) Si $f(t) = 1$ entonces $F(s) \bullet G(s) = G(s)$ d) ninguna de las anteriores
3.	Sea $f(t) = t$ si $t \in [0,1]$ y $f(t) = 0$ si $t \notin [0,1]$ Entonces: a) No existe transformada de Laplace de f(t) b) $\mathcal{L}[f(t)] = \frac{1}{s^2}$ c) $\mathcal{L}[f(t)] = \frac{1}{s^2}(1-e^{-s}) - \frac{e^{-s}}{s}$ d) $\mathcal{L}[f(t)] = \frac{1}{s^2}(1-e^{-s})$
4.	La solución de la ecuación: $y''(t) - 2y'(t) + 5y(t) = 3e^{t}\,\mathrm{sen}(t)$ con $y(0) = 0 \wedge y'(0) = 1$ es: a) $y(t) = e^{-2t}[\cos(t) + \mathrm{sen}(t)]$ b) $y(t) = e^{2t}\,\mathrm{sen}(t)$ c) $y(t) = e^{t}\,\mathrm{sen}(t)$ d) ninguna de las anteriores
5.	La antitransformada de Laplace de : $Y(s) = 2(s^2+1)^{-2}$ es: a) $y(t) = 2\,\mathrm{sen}^2(t)$ b) $y(t) = \mathrm{sen}(t) - t\cos(t)$ c) $y(t) = 2\,t\,\mathrm{sen}(t)$ d) ninguna de las anteriores
6.	El valor de la integral $\int_0^{\infty} \frac{e^{-2t} - e^{-6t}}{t}\,dt$ es: a) 0 b) ln(2) c) ln(3) d) ninguna de las anteriores

7.	La Transformada de Laplace de $f(t) = 3\,\delta(t) + t$ cuando $s \to \infty$, tiende a: a) cero b) tres c) infinito d) ninguna de las anteriores
8.	Sea $F(s) = \mathcal{L}[f(t)]$ tal que $F(s) = \dfrac{3s^2+9}{2s^3-23s^2+43s-42}$ El valor de f(0) es: a) 0 b) 3/2 c) 3 d) infinito
9.	El valor de la integral $\int_0^\infty \operatorname{sen}(2t)t^2 e^{-3t}\,dt$ es: a) 0 b) $\dfrac{2}{13}$ c) $\dfrac{92}{2197}$ d) ninguna de las anteriores
10.	La Transformada de Laplace de $f(t) = -5\,e^{2t} + 2\cos(t)\,e^{3t} - 19\operatorname{sen}(t)\,e^{3t}$ es: a) $\dfrac{-3s^2+s}{(s-2)(s^2-6s+10)}$ b) $\dfrac{s^2-4s-5}{s^3-7s^2+17s+25}$ c) $\dfrac{6s+3}{s^3+4s^2+13s}$ d) ninguna de las anteriores
11.	El valor de la integral: $\int_0^\infty e^{-t}\dfrac{\operatorname{sen}(t)}{t}\,dt$ es: a) 0 b) $\dfrac{1}{2}$ c) $\dfrac{\pi}{4}$ d) infinito

Transformadas de Laplace

PARTE 1 transformada directa de Laplace - propiedades

① Calcule las transformadas de Laplace de las sig. funciones, utilizando la tabla y la prop. de linealidad:

a) $y(t) = 3+5t-2t^2+t^3 \;;\; t>0$

$$F(s) = \frac{3}{s} + \frac{5}{s^2} - 2\cdot\frac{2!}{s^{2+1}} + \frac{3!}{s^{3+1}} \longrightarrow \boxed{F(s) = \frac{3}{s} + \frac{5}{s^2} - \frac{4}{s^3} + \frac{6}{s^4}}$$

b) $y(t) = \frac{1}{4}\left(e^{4t}-1\right) \;;\; t>0$

$$F(s) = \frac{1}{4}\left(\frac{1}{s-4} - \frac{1}{s}\right) = \frac{1}{4}\left(\frac{s-(s-4)}{(s-4)\,s}\right) = \frac{1}{4}\cdot\left(\frac{\cancel{s}-\cancel{s}+4}{(s-4)\,s}\right) = \frac{4}{4\,(s-4)\,s} = \boxed{\frac{1}{(s-4)\,s} = F(s)}$$

c) $y(t) = \frac{1}{a^2}\left(e^{at}-at-1\right) \;;\; t>0$

$$F(s) = \frac{1}{a^2}\left(\frac{1}{s-a} - \frac{a}{s^2} - \frac{1}{s}\right) = \frac{1}{a^2}\left(\frac{s^2-(s-a)a-(s-a)s}{(s-a)\,s^2}\right) =$$

$$= \frac{1}{a^2}\left(\frac{\cancel{s^2}-\cancel{as}+a^2-\cancel{s^2}+\cancel{as}}{(s-a)\,s^2}\right) = \frac{\cancel{a^2}}{\cancel{a^2}(s-a)\,s^2} \longrightarrow \boxed{F(s) = \frac{1}{(s-a)\,s^2}}$$

d) $y(t) = 4\cos(3t) - 5\,\mathrm{sen}(2t) \;;\; t>0$

$$F(s) = 4\cdot\frac{s}{9+s^2} - 5\,\frac{2}{4+s^2} \longrightarrow \boxed{F(s) = \frac{4s}{9+s^2} - \frac{10}{4+s^2}}$$

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② Utilizando las prop. de la transformada de Laplace halle $\mathcal{L}[f(t)]$

a) $f(t) = t^2 \underbrace{e^{4t}}$; $t>0$ $g(t) = t^2 \rightarrow G(s) = \frac{2}{s^3}$

traslación en el dominio: $a = 4$

$$\mathcal{L}\left[t^2 e^{4t}\right] = F(s-4) = \frac{2}{(s-4)^3} \rightarrow \boxed{\mathcal{L}\left[t^2 e^{4t}\right] = \frac{2}{(s-4)^3}}$$

b) $f(t) = 5\, e^{-5t} \cos(3t)$; $t>0$

traslación en el dominio: $a = -5$; $g(t) = \cos(3t) \rightarrow G(s) = \frac{s}{9+s^2}$

$$\mathcal{L}\left[5\, e^{-5t} \cos(3t)\right] = 5\, G(s+5) = 5 \cdot \frac{s+5}{9+(s+5)^2} = \frac{5s+25}{9+s^2+10s+25}$$

$$\boxed{\mathcal{L}\left[5 . e^{-5t} \cos(3t)\right] = \frac{5s+25}{s^2+10s+34}}$$

c) $f(t) = \underbrace{e^{-t}}_{\text{trasl. en el dominio}} t \,\text{sen}(2t)$; $t>0$ $\longrightarrow F(s-a)$

tomo $g(t) = t \underbrace{\text{sen}(2t)}_{h(t)} \longrightarrow G(s) = -1 \cdot H'(s)$

$$\rightarrow H(s) = \frac{2}{4+s^2} = 2(4+s^2)^{-1} \rightarrow H'(s) = -2(4+s^2)^{-2}\, 2s$$

$$\boxed{H'(s) = \frac{-4s}{(4+s^2)^2}}$$

$$G(s) = -1 \cdot H'(s) = \frac{4s}{(4+s^2)^2}$$

$$\rightarrow F(s) = G(s+1) = \frac{4(s+1)}{(4+(s+1)^2)} = \boxed{\frac{4s+4}{(4+(s+1)^2)^2} = F(s)}$$

d) $f(t) = (t-\boxed{1})^4 \; ; \; t > \boxed{1}$

$g(t) = t^4 \quad , \; g(t-1) = f(t) = (t-1)^4 \rightarrow \boxed{f(t) = g(t-1) \quad t > 1}$

Segunda propiedad de traslación

$\rightarrow \mathcal{L}[f(t)] = \mathcal{L}[g(t-1)] = e^{-1s} \cdot G(s) = \boxed{e^{-s} \dfrac{24}{s^5} = F(s)}$ ✓

$G(s) = \dfrac{4!}{s^{4+1}} = \dfrac{24}{s^5} = G(s)$

e) $f(t) = e^{-2t} \operatorname{sen}(2t) + t^2 e^{3t} \quad ; \; t > 0$

linealidad + traslación en el dominio.

- $f_1(t) = e^{-2t} \underbrace{\operatorname{sen}(2t)}_{g_1(t)} \rightarrow \mathcal{L}[f_1(t)] = G_1(s+2) = \dfrac{2}{4+(s+2)^2} = \dfrac{2}{s^2+4s+8}$

$\rightarrow G_1(s) = \dfrac{2}{4+s^2}$

- $f_2(t) = \underbrace{t^2}_{g_2(t)} e^{3t} \rightarrow \mathcal{L}[f_2(t)] = G_2(s-3) = \dfrac{2}{(s-3)^3}$

$\rightarrow G_2(s) = \dfrac{2!}{s^{2+1}} = \dfrac{2}{s^3}$

$$\boxed{F(s) = \frac{2}{4+(s+2)^2} + \frac{2}{(s-3)^3}}$$ ✓

f) $f(t) = \underbrace{2t^2 e^{-t}}_{\text{traslación en el dom.}} \underbrace{- t + \cos(4t)}_{\text{linealidad}}$

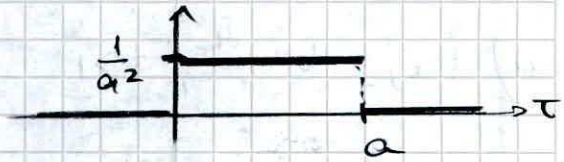
$g(t) = 2\underbrace{t^2}_{h(t)} e^{-t} \rightarrow h(t) = 2t^2 \rightarrow H(s) = 2 \cdot \dfrac{2!}{s^3} = \dfrac{4}{s^3} \longrightarrow G(s) = H(s+1) =$

$= \dfrac{4}{(s+1)^3}$

$$\boxed{F(s) = \frac{4}{(s+1)^3} - \frac{1}{s^2} + \frac{s}{16+s^2}}$$ ✓

③ Calcule las transformadas de Laplace aplicando adecuadamente alguna propiedad conveniente o algún paso algebraico:

a) $y(t) = \frac{1}{a^2}$; $0 < t < a$



$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt = \int_0^a \frac{1}{a^2} e^{-st}\, dt = \frac{1}{a^2} \cdot \frac{e^{-st}}{-s}\Big|_0^a =$$

$$= \frac{1}{a^2}\left(\frac{e^{-as}}{-s} - \frac{1}{-s}\right) = \boxed{\frac{1 - e^{-as}}{s a^2} = F(s)}$$

b) $y(t) = sen(5t + \pi/3)$; $t > 0$ teorema del valor inicial

* funciones de la suma o dif de ángulos

① $sen(x+y) = sen(x)\cos(y) + sen(y)\cos(x)$

$$y(t) = sen(\overset{x}{5t} + \overset{y}{\pi/3}) = sen(5t)\cos(\pi/3) + sen(\pi/3)\cos(5t) =$$

$$= sen(5t)\ 1/2 + \frac{\sqrt{3}}{2}\cos(5t)$$

$$\boxed{Y(s) = \frac{5}{2(s^2+25)} + \frac{\sqrt{3}\, s}{2(s^2+25)}}$$

c) $y(t) = \cos^3(t)$; $t > 0$

función potencia

$$\cos^2(t) \overset{\downarrow}{=} \frac{1}{2}\left(1 + \cos(2t)\right)$$

$$y(t) = \cos(t)\cos^2(t) = \cos(t)\,\frac{1}{2}\cdot(1 + \cos(2t)) = \frac{1}{2}\left(\cos(t) + \cos(t)\cos(2t)\right) =$$

tomo $x = t$, $y = 2t$

$$\overset{⑥}{=} \frac{1}{2}\Big[\cos(t) + \frac{1}{2}\cdot 2\cdot \underbrace{\cos(t)\cos(2t)}_{\text{ver tabla ⑥}}\Big] = \frac{1}{2}\Big[\cos(t) + \frac{1}{2}\big(\cos(\overset{x-y}{t-2t}) + \cos(t+2t)\big)\Big] =$$

$$= \frac{1}{4}\Big[2\cos(t) + \overset{=\cos(t)}{\cos(-t)} + \cos(3t)\Big] = \frac{1}{4}\left(3\cos(t) + \cos(3t)\right)$$

linealidad:

$$\boxed{Y(s) = \frac{3s}{4(1+s^2)} + \frac{s}{4(9+s^2)}}$$

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T. Laplace

d) $y(t) = a^t$; $t > 0$; $a \in \mathbb{R}^+$

$$a^t = e^{\ln(a^t)} = e^{t\ln(a)} = y(t) \rightarrow \boxed{Y(s) = \frac{1}{s - \ln(a)}}$$ ✓

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

e) $y(t) = (t-1)^4$, $t > 0$

$$y(t) = t^4 - 4t^3 + 6t^2 - 4t + 1$$

binomio de Newton y linealidad.

$$Y(s) = \frac{4!}{s^5} - \frac{4 . 3!}{s^4} + \frac{6 . 2!}{s^3} - \frac{4}{s^2} + 1 = \boxed{\frac{24}{s^5} - \frac{24}{s^4} + \frac{12}{s^3} - \frac{4}{s^2} + 1 = Y(s)}$$ ✓

f) $y(t) = \text{sen}(5t)\cos(2t)$; $t > 0$

$$\text{sen}(x) + \text{sen}(y) = 2\,\text{sen}\left(\frac{x+y}{2}\right) . \cos\left(\frac{x-y}{2}\right)$$

busco valores de x e y tales que $\frac{x+y}{2} = 5t$ y $\frac{x-y}{2} = 2t$

$$\begin{cases} x+y = 10t \\ x-y = 4t \end{cases} \rightarrow \begin{array}{l} x = 7t \\ y = 3t \end{array}$$

$$y(t) = \frac{1}{2} . 2 . \text{sen}(5t)\cos(2t) = \frac{1}{2}\, 2\,\text{sen}\left(\frac{7+3}{2}t\right)\cos\left(\frac{7-3}{2}t\right) =$$

$$= \frac{1}{2}\left(\text{sen}(7t) + \text{sen}(3t)\right) \rightarrow \text{linealidad.}$$

$$Y(s) = \frac{1}{2}\left[\frac{7}{49+s^2} + \frac{3}{9+s^2}\right] = \frac{1}{2} . \frac{7(9+s^2) + 3(49+s^2)}{(49+s^2)(9+s^2)} =$$

$$= \frac{1}{2}\,\frac{63 + 7s^2 + 147 + 3s^2}{(49+s^2)(9+s^2)} = \frac{1}{2}\,\frac{210 + 10s^2}{(49+s^2)(9+s^2)} = \boxed{\frac{105 + 5s^2}{(49+s^2)(9+s^2)} = Y(s)}$$ ✓

Sylvina

(4) Sea $f(t) = k \operatorname{sen}\left(2t + \frac{\pi}{6}\right)$. Determine el valor de k sabiendo que $\lim_{s \to \infty} sF(s) = 4$ (Recuerde el teorema de valor inicial)

$$\lim_{t \to 0} k \operatorname{sen}\left(2t + \frac{\pi}{6}\right) = \lim_{s \to \infty} sF(s) = 4$$

$$\lim_{t \to 0} k \operatorname{sen}\left(2t + \frac{\pi}{6}\right) = k \left(\operatorname{sen} \frac{\pi}{6}\right) = k \frac{1}{2} = 4 \rightarrow \boxed{k = 8}$$

PARTE 2: Antitransformada de Laplace - propiedades

(5) Calcule las antitransformadas de Laplace de las sig. func.:

a) $$Y(s) = \frac{25}{s^3} = a \cdot \frac{n!}{\underbrace{s^{n+1}}_{3 \to n=2}} = \overbrace{a \cdot \frac{2}{s^3}}^{25} = \frac{25}{2} \frac{2}{s^3} \rightarrow \boxed{y(t) = \frac{25}{2} t^2}$$ ✓

t^n

b) $$Y(s) = \frac{8}{s^2+3} + \frac{1}{s}$$

$$\frac{8}{s^2+3} = \frac{\sqrt{3}}{s^2 + (\sqrt{3})^2} \cdot a \Rightarrow 8 = a\sqrt{3} \rightarrow a = \frac{8}{\sqrt{3}} \rightarrow \boxed{y(t) = \frac{8}{\sqrt{3}} \operatorname{sen}(\sqrt{3}t) + 1}$$ ✓

c) $$Y(s) = \frac{12}{s-3} - \frac{8s}{s^2+4}$$

$$\frac{12}{s-3} = 12 \cdot \frac{1}{s-3} \rightarrow y_1(t) = 12e^{3t} \; ; \; \frac{8s}{s^2+4} \rightarrow y_2(t) = 8(\cos 2t)$$

$$\boxed{Y(t) = 12e^{3t} - 8\cos(2t)}$$ ✓

d) $$Y(s) = \frac{-1}{s+3} \rightarrow \boxed{y(t) = -e^{-3t}}$$ ✓

e) $$Y(s) = \frac{6}{(s-1)^4} = \frac{3!}{(s-1)^{3+1}} \longrightarrow \boxed{y(t) = e^t \cdot t^3}$$ ✓

f) $$Y(s) = \frac{s+3}{\underbrace{s^2+6s+13}_{(s+3)^2 = s^2+6s+9}} = \overbrace{\frac{s+3}{(s+3)^2+4}}^{\text{despl. en el dom.}} \rightarrow \boxed{y(t) = e^{-3t} \cos(2t)}$$ ✓

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g)
$$Y(s) = \frac{s-1}{s^2+2s+2}$$

$$Y(s) = \frac{s-1}{(s+1)^2+1} = \frac{\overbrace{s+1-2}^{-1}}{(s+1)^2+1} = \frac{s+1}{(s+1)^2+1} - \frac{2}{(s+1)^2+1}$$

$$\boxed{y(t) = e^{-t} \cdot (\cos(t) - 2\,\mathrm{sen}(t))}$$

h)
$$Y(s) = \frac{4s}{(s^2+4)^2}$$

si tengo $f(t) = t\, g(t) \rightarrow F(s) = -1 \cdot G'(s)$

si considero $g(t) = \mathrm{sen}(2t) \rightarrow G(s) = \frac{2}{s^2+4} \rightarrow G'(s) = \frac{-2 \cdot 2s}{(s^2+4)^2}$

$$\rightarrow F(s) = -1 \cdot \frac{-4s}{(s^2+4)^2}$$

$$\boxed{\rightarrow y(t) = t \cdot \mathrm{sen}(2t)}$$

Sig

⑥ Calcule las antitransformadas de Laplace de las sig. funciones previamente separando en fracciones simples:

a) $$F(s) = \frac{s+7}{s^2-6s+5} = \frac{s+7}{(s-5)(s-1)} = \frac{A}{(s-5)} + \frac{B}{(s-1)} = \frac{A(s-1)+B(s-5)}{(s-5)\ (s-1)}$$

2 valores reales ↗ $s=5$ y $s=1$

$$= \frac{s(A+B) + (-A-5B)}{(s-5)\ (s-1)} = \frac{s+7}{(s-5)(s-1)} \rightarrow \begin{cases} A+B=1 \\ -A-5B=7 \end{cases} \rightarrow \boxed{\begin{matrix} A=3 \\ B=-2 \end{matrix}}$$

$$F(s) = \frac{3}{s-5} + \frac{-2}{s-1} \rightarrow \boxed{f(t) = 3e^{5t} - 2e^{t}}$$ ✓

b) $$F(s) = \frac{s^2+2s+2}{s^2+3s+2} = \frac{s^2+2s+2+s-s}{s^2+3s+2} = \frac{s^2+3s+2-s}{s^2+3s+2} =$$

$$= \frac{s^2+3s+2}{s^2+3s+2} - \frac{s}{s^2+3s+2} = 1 - \frac{s}{s^2+3s+2} = 1 - \frac{s}{(s+1)\ (s+2)} =$$

$$= 1 - \frac{A}{s+1} - \frac{B}{s+2} = 1 - \frac{A(s+2)+B(s+1)}{(s+1)\ (s+2)} \rightarrow \begin{cases} A+B=1 \\ 2A+B=0 \end{cases} \rightarrow \begin{matrix} A=-1 \\ B=2 \end{matrix}$$

$$F(s) = 1 + \frac{1}{s+1} - \frac{2}{s+2} \rightarrow \boxed{f(t) = \delta(t) + e^{-t} - 2e^{-2t}}$$ ✓

c) $$F(s) = \frac{1}{(s+2)^2(s+1)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s+2)^2} = \frac{A(s^2+4s+4) + \overbrace{Bs(s+1)}^{Bs^2+Bs} + C(s+1)}{(s+1)\ (s+2)^2}$$

$$\rightarrow \begin{matrix} s^2: \\ s: \\ N^\circ: \end{matrix} \begin{cases} A+B=0 \\ 4A+B+C=0 \\ 4A+C=1 \end{cases} \rightarrow \begin{matrix} A=1 \\ B=-1 \\ C=-3 \end{matrix} \rightarrow F(s) = \underbrace{\frac{1}{s+1}}_{f_1(t)=e^{-t}} - \frac{\overbrace{s+3}^{s+2+1}}{(s+2)^2} =$$

$$= \frac{1}{s+1} - \cancel{\frac{s+2}{(s+2)^2}} - \frac{1}{(s+2)^2} = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$\boxed{f(t) = e^{-t} - e^{-2t} - te^{-2t}}$$ ✓

$$\Big(\underbrace{\frac{1}{s+2}}_{G(s)}\Big)' = \left((s+2)^{-1}\right)' = \underbrace{-\frac{1}{(s+2)^2}}_{G'(s)}$$

$g(t) = e^{-2t}$

$t\,e^{2t}$

d) $F(s) = \dfrac{s^2+9s+19}{(s+1)\ (s+2)\ (s+4)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+4} =$

$= \dfrac{A(s+2)(s+4) + B(s+1)(s+4) + C\ (s+1)\ (s+2)}{(s+1)\ (s+2)\ (s+4)} =$

$= \dfrac{A(s^2+6s+8) + B(s^2+5s+4) + C\ (s^2+3s+2)}{(s+1)\ (s+2)\ (s+3)}$

$\rightarrow \begin{cases} s^2 : A+B+C = 1 \\ s : 6A+5B+3C = 9 \\ \# : 8A+4B+2C = 19 \end{cases} \longrightarrow \left. \begin{array}{l} A = \frac{11}{3} \\ B = \frac{-5}{2} \\ C = -\frac{1}{6} \end{array} \right\} F(s) = \dfrac{11}{3(s+1)} - \dfrac{5}{2(s+2)} - \dfrac{1}{6(s+4)}$

$$\boxed{f(t) = \frac{11}{3} e^{-t} - \frac{5}{2} e^{-2t} - \frac{1}{6} e^{-4t}}$$

e) $F(s) = \dfrac{2e^{-0,5s}}{s^2+6s+3} \rightarrow$ desploz. en t $\rightarrow f(t) = g(t-0,5)$ I

$G(s) = \dfrac{2}{s^2+6s+3} = \dfrac{2}{(s+3-\sqrt{6})\ (s+3+\sqrt{6})} =$

$= \dfrac{A(s+3+\sqrt{6}) + B(s+3-\sqrt{6})}{(s+3-\sqrt{6})\ (s+3+\sqrt{6})} \rightarrow \begin{cases} A+B = 0 \\ (3+\sqrt{6})A + (3-\sqrt{6})B = 2 \end{cases}$

$A = \underbrace{0,408}_{\frac{1}{\sqrt{6}}}$ $\quad B = \underbrace{-0,408}_{-\frac{1}{\sqrt{6}}}$

$G(s) = \dfrac{1}{\sqrt{6}} \cdot \dfrac{1}{(s+(3-\sqrt{6}))} - \dfrac{1}{\sqrt{6}} \dfrac{1}{s+(3+\sqrt{6})}$

$\rightarrow g(t) = \dfrac{1}{\sqrt{6}} e^{-(3-\sqrt{6})t} - \dfrac{1}{\sqrt{6}} e^{-(3+\sqrt{6})t}$

I $$\boxed{f(t) = \frac{1}{\sqrt{6}} e^{-(3-\sqrt{6})(t-0,5)} - \frac{1}{\sqrt{6}} e^{-(3+\sqrt{6})(t-0,5)}}$$

f) $$F_{(s)} = \frac{6s^2-13s+12}{s(s-1)(s-6)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-6} = \frac{A\overbrace{(s-1)(s-6)}^{s^2-7s+6}+B(s^2-6s)+C(s^2-s)}{s(s-1)(s-6)}$$

$$\rightarrow \begin{cases} s^2: \ A+B+C = 6 \\ s: \ -7A-6B-C = -13 \\ \#: \ 6A = 12 \end{cases} \rightsquigarrow \begin{matrix} A=2 \\ B=-1 \\ C=5 \end{matrix} \rightsquigarrow F_{(s)} = \frac{2}{s} - \frac{1}{s-1} + \frac{5}{s-6}$$

$$\boxed{f_{(t)} = 2 - e^{t} + 5e^{6t}}$$ ✓

g) $$s^2F_{(s)} - 4F_{(s)} = \frac{s}{s+1}$$

$$F_{(s)}(s^2-4) = \frac{s}{s+1} \longrightarrow F_{(s)} = \frac{s}{(s^2-4)(s+1)} = \frac{s}{(s+2)(s-2)(s+1)} =$$

$$= \frac{A(s-2)(s+1)+B(s+2)(s+1)+C(s+2)(s-2)}{(s+2)(s-2)(s+1)} =$$

$$= \frac{A(s^2-s-2)+B(s^2+3s+2)+C(s^2-4)}{(s+2)(s-2)(s+1)} \rightsquigarrow \begin{cases} A+B+C = 0 \\ -A+3B = 1 \\ -2A+2B-4C = 0 \end{cases}$$

$$A = -\frac{1}{2} \quad B = \frac{1}{6} \quad C = \frac{1}{3}$$

$$F_{(s)} = -\frac{1}{2(s+2)} + \frac{1}{6(s-2)} + \frac{1}{3(s+1)}$$

$$\rightsquigarrow \boxed{f_{(t)} = -\frac{1}{2}e^{-2t} + \frac{1}{6}e^{2t} + \frac{1}{3}e^{-t}}$$ ✓

h) $$F_{(s)} = \frac{7s^2-41s+84}{(s-1)(s^2-4s+13)} = \frac{A}{(s-1)} + \frac{Bs+C}{(s^2-4s+13)} =$$

$$(s-2)^2+9$$
$$s^2-4s+4+9$$

$$= \frac{A(s^2-4s+13)+B(s^2-s)+C(s-1)}{(s-1)(s^2-4s+13)} \longrightarrow \begin{cases} A+B = 7 \\ -4A-B+C = -41 \\ 13A - C = 84 \end{cases} \begin{matrix} A=5 \\ B=2 \\ C=-19 \end{matrix}$$

$$F_{(s)} = \frac{5}{s-1} + \frac{2s-19}{(s-2)^2+9} = \frac{5}{s-1} + \frac{\overbrace{2s-4}^{2(s-2)}-15}{(s-2)^2+9} = \frac{5}{s-1} + \frac{2(s-2)}{(s-2)^2+9} - \frac{\overbrace{15}^{5\times 3}}{(s-2)^2+9}$$

$$\rightarrow \boxed{f_{(t)} = 5e^{t} + 2\cos(3t)\,e^{2t} - 5\,\text{sen}(3t)\,e^{2t}}$$ ✓

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i) $F(s) = \dfrac{-2s^2+8s-14}{(s+1)(s^2-2s+5)} = \dfrac{A(s^2-2s+5)+B(s^2+s)+C(s+1)}{(s+1)[(s-1)^2+4]}$

$$\begin{cases} A+B = -2 \\ -2A+B+C = 8 \\ 5A + C = -14 \end{cases} \quad \begin{matrix} A=-3 \\ B=1 \\ C=1 \end{matrix} \quad F(s) = \frac{-3}{(s+1)} + \frac{\overbrace{s+1}^{s-1+2}}{(s-1)^2+4}$$

$$\to F(s) = \frac{-3}{s+1} + \frac{s-1}{(s-1)^2+4} + \frac{2}{(s-1)^2+4}.$$

$$\to \boxed{f(t) = -3e^{-t} + e^{t}.\cos(2t) + 2e^{t}\,\text{sen}(2t)}$$

j) $F(s) = \dfrac{(4s+2)\,e^{-2s}}{(s-1)(s+2)}$ G despl. en el tiempo $\to f(t) = g(t-2)$ (I)

tomo: $G(s) = \dfrac{4s+2}{(s-1)(s+2)} = \dfrac{A(s+2)+B(s-1)}{(s-1)(s+2)} \to \begin{cases} A+B = 4 \\ 2A-B = 2 \end{cases} \begin{matrix} A=2 \\ B=2 \end{matrix}$

$$\to G(s) = \frac{2}{s-1} + \frac{2}{s+2} \longrightarrow \boxed{g(t) = 2e^{t} + 2e^{-2t}}$$

(I) $\boxed{f(t) = 2e^{(t-2)} + 2e^{-2t+4}}$

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(7)

a) Aplicando el teorema de valor Final, halle el valor de f(t) para $t \to \infty$, siendo:

$F(s) = \dfrac{10}{s^2+s}$. Verifique el resultado obtenido.

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s) = \lim_{s\to 0} \frac{\cancel{s}\cdot 10}{\cancel{s}\,\underbrace{(s+1)}_{\to 0}} = 10 \checkmark$$

b) Dada: $F(s) = \dfrac{1}{(s+2)^2}$ halle f(0) por el teorema del valor inicial. Verifique los valores obtenidos

$$\text{TVI} \quad \lim_{t\to 0} f(t) = \lim_{s\to\infty} s\,F(s) = \lim_{s\to\infty} \frac{s}{(s+2)^2} = \lim_{s\to\infty} \frac{s}{s^2+4s+4} =$$

$$= \lim_{s\to\infty} \frac{\cancel{s}^{\,1}}{\cancel{s}\,(\underbrace{s}_{\to\infty}+4+\underbrace{4/s}_{\to 0})} = 0 \checkmark$$

$$F(s) = \frac{1}{(s+2)^2} \longrightarrow f(t) = t\,e^{-2t} \to f(0) = 0 \checkmark$$

c) Dada $F(s) = \dfrac{2s+1}{s^2+2s}$, halle el valor de f(t) para:

i) $t = 0$

$$\times\text{TVI}: \lim_{t\to 0} f(t) = \lim_{s\to\infty} s\,F(s) = \lim_{s\to\infty} \frac{2s^2+s}{s^2+2s} = \lim_{s\to\infty} \frac{s^2(2+\overbrace{1/s}^{\to 0})}{s^2(1+\underbrace{2/s}_{\to 0})} = 2$$

$$F(s) = \frac{2s+1}{s^2+2s} = \frac{2s+1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} \to \begin{cases} A+B = 2 \\ 2A = 1 \end{cases} \to \begin{matrix} A = 1/2 \\ B = 3/2 \end{matrix}$$

$$F(s) = \frac{1/2}{s} + \frac{3/2}{s+2} \to f(t) = \frac{1}{2} + \frac{3}{2}e^{-2t} \to f(0) = \frac{1}{2} + \frac{3}{2} = 2 \checkmark$$

ii) $t = \infty$

$$\lim_{t\to\infty} f(t) = \lim_{t\to\infty} \frac{1}{2} + \frac{3}{2}\overbrace{e^{\overbrace{-2t}^{\to -\infty}}}^{\to 0} = \frac{1}{2} \checkmark$$

iii) $t = 0{,}5$

$$f(0{,}5) = \frac{1}{2} + \frac{3}{2}e^{-2\cdot\frac{1}{2}} = \frac{1}{2} + \frac{3}{2}e^{-1} \checkmark$$

Mat sup.
t. Laplace

(8) Resuelva las siguientes Ecuaciones Diferenciales por transformada de Laplace:

a) $2x''(t) + 7x'(t) + 3x(t) = 6$ con $x(0) = 0 \wedge x'(0) = 0$

$$2\left[s^2 X(s) - s\overset{0}{x(0)} - \overset{0}{x'(0)}\right] + 7\left[s X(s) - \overset{0}{x(0)}\right] + 3X(s) = \frac{6}{s}$$

$$2s^2 X(s) + 7s X(s) + 3X(s) = \frac{6}{s}$$

$$X(s)(2s^2 + 7s + 3) = \frac{6}{s}$$

$$X(s) = \frac{6}{s(2s^2+7s+3)} = \frac{\cancel{6}\,3}{s\cdot\cancel{2}(s+1/2)(s+3)} =$$

$$= \frac{A\overbrace{(s+1/2)(s+3)}^{s^2+7/2s+3/2} + B(s^2+3s) + C(s^2+s/2)}{s(s+1/2)(s+3)} =$$

$$\begin{cases} A+B+C = 0 \\ 7/2A + 3B + C/2 = 0 \\ 3/2A = 3 \end{cases} \rightarrow \begin{cases} C = 2/5 \\ B = -12/5 \\ A = 2 \end{cases}$$

$$Y(s) = \frac{2}{s} - \frac{12/5}{s+1/2} + \frac{2/5}{s+3}$$

$$\boxed{y(t) = 2 - \frac{12}{5}e^{-\frac{t}{2}} + \frac{2}{5}e^{-3t}}$$ ✓

b) $x''(t) + 3x'(t) + 6x(t) = 2$ con $x(0) = 0 \wedge x'(0) = 0$

$$s^2 X(s) - s\overset{0}{x(0)} - \overset{0}{x'(0)} + 3\left[s X(s) - \overset{0}{x(0)}\right] + 6X(s) = \frac{2}{s}$$

$$X(s)(s^2 + 3s + 6) = \frac{2}{s}$$

$$X(s) = \frac{2}{s(s^2+3s+6)} = \frac{A}{s} + \frac{Bs+C}{(s+3/2)^2+\frac{15}{4}}$$

$$\rightarrow A(s^2+3s+6) + Bs^2 + Cs = 2$$

$$\rightarrow \begin{cases} A+B = 0 \\ 3A + C = 0 \\ 6A = 2 \end{cases} \quad \begin{cases} A = 1/3 \\ B = -1/3 \\ C = -1 \end{cases}$$

$$\rightarrow Y(s) = \frac{1/3}{s} - \frac{1/3 s + 1}{(s+3/2)^2 + 15/4} =$$

$$Y(s) = \frac{1/3}{s} - \frac{1/3 s + 1/3\cdot 3/2 + 1/2}{(s+3/2) + 15/4} =$$

$$= \frac{1/3}{s} - \frac{1/3(s+3/2)}{(s+3/2)^2+15/4} - \frac{1/2\cdot\frac{\sqrt{15}}{\sqrt{15}}}{(s+3/2)^2+15/4} \longrightarrow \boxed{y(t) = \frac{1}{3} - \frac{1}{3}\cos\left(\frac{\sqrt{15}}{2}\right)e^{-\frac{3}{2}t} - \frac{1}{\sqrt{15}}\operatorname{sen}\left(\frac{\sqrt{15}}{2}\right)e^{-\frac{3}{2}t}}$$ ✓

Sw

c) $3y'(t) + 5y(t) = 6$ con $y(0) = 0$

$$3\left[sY(s) - \overset{0}{y(0)}\right] + 5Y(s) = \frac{6}{s}$$

$$Y(s)\cdot(3s+5) = \frac{6}{s} \rightarrow Y(s) = \frac{\cancel{6}\,2}{s\,\cancel{3}\left(s+\frac{5}{3}\right)} = \frac{A}{s} + \frac{B}{(s+5/3)}$$

$$As + 5/3A + Bs = 2 \quad \left.\begin{matrix} A+B=0 \\ 5/3A = 2 \end{matrix}\right\} \begin{matrix} A = 6/5 \\ B = -6/5 \end{matrix} \Bigg\} Y(s) = \frac{6/5}{s} - \frac{6/5}{s+5/3} \rightarrow \boxed{y(t) = \frac{6}{5} - \frac{6}{5}e^{-\frac{5}{3}t}}$$ ✓

d) $x'(t) + x(t) = e^{-2t}$ con $x(0) = 0$

$$sX(s) - \overset{0}{x(0)} + X(s) = \frac{1}{s+2}$$

$$X(s)\,(s+1) = \frac{1}{s+2} \longrightarrow X(s) = \frac{1}{(s+1)\,(s+2)} = \frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix} \longrightarrow X(s) = \frac{1}{s+1} - \frac{1}{s+2} \longrightarrow \boxed{x(t) = e^{-t} - e^{-2t}}$$ ✓

e) $y''(t) + 4y'(t) + 4y(t) = \overbrace{\underset{-15e^{-5t}\ +\ 2e^{-5t}}{3x'(t) + 2x(t)}}^{-13e^{-5t}}$ con $x(t) = e^{-5t} \rightarrow x'(t) = -5e^{-5t}$

$y(0) = 0$

$y'(0) = 0$

$$s^2Y(s) - s\overset{0}{y(0)} - \overset{0}{y'(0)} + 4\left[sY(s) - \overset{0}{y(0)}\right] + 4Y(s) = -\frac{13}{s+5}$$

$$Y(s)\,(s^2+4s+4) = \frac{-13}{(s+5)\,(s^2+4s+4)} = \frac{A}{s+5} + \frac{Bs+C}{\underbrace{s^2+4s+4}_{(s+2)^2}}$$

$$A(s^2+4s+4) + B(s^2+5s) + C(s+5) = -13$$

$$\left.\begin{cases} A+B = 0 \\ 4A+5B+C = 0 \\ 4A+5C = -13 \end{cases}\right\} \rightarrow \left.\begin{matrix} A = -13/9 \\ B = 13/9 \\ C = -13/9 \end{matrix}\right\} \rightarrow Y(s) = \frac{-13/9}{s+5} + \frac{13/9\,s - 13/9}{(s+2)^2} = \frac{-13/9}{s+5} + \frac{13/9\,(\overbrace{s-1+2-2}^{s+2-3})}{(s+2)^2} =$$

$$= \frac{-13/9}{s+5} + \frac{13/9\,(s+2)}{(s+2)^2} + \frac{13/9_3\,(-\cancel{3})}{(s+2)^2} =$$

$$\rightarrow Y(s) = \frac{-13/9}{s+5} + \frac{13/9}{s+2} - \frac{13/3}{(s+2)^2} \longrightarrow \boxed{y(t) = \frac{-13}{9}e^{-5t} + \frac{13}{9}e^{-2t} - \frac{13}{3}te^{-2t}}$$ ✓

sin desp.

$$\frac{13/3}{s^2} \rightarrow \frac{13}{3}t$$

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f) $y''(t) + 6\,y'(t) + 18\,y(t) = 13\,e^{-5t}$ $\qquad y(0) = 1 \ \wedge \ y'(0) = -2$

$$s^2\,Y(s) - s\,\overset{1}{y(0)} - \overset{-2}{y'(0)} + 6\left[s\,Y(s) - \overset{1}{y(0)}\right] + 18\,Y(s) = \frac{13}{s+5}$$

$$s^2\,Y(s) - s + 2 + 6s\,Y(s) - 6 + 18\,Y(s) = \frac{13}{s+5}$$

$$Y(s)\,(s^2+6s+18) = \frac{13}{s+5} + s + 4 = \frac{13 + \overbrace{(s+4)(s+5)}^{s^2+9s+20}}{s+5} = \frac{s^2+9s+33}{s+5}$$

$$Y(s) = \frac{s^2+9s+33}{(s+5)\,(s^2+6s+18)} = \frac{s^2+9s+33}{(s+5)\,((s+3)^2+9)} = \frac{A}{s+5} + \frac{Bs+C}{(s+3)^2+9}$$

$$\rightarrow A\,(s^2+6s+18) + B\,(s^2+5s) + C\,(s+5) = s^2+9s+33 \rightarrow \begin{cases} A + B = 1 \\ 6A + 5B + C = 9 \\ 18A + 5C = 33 \end{cases} \begin{matrix} A = 1 \\ B = 0 \\ C = 3 \end{matrix}$$

$$Y(s) = \frac{1}{s+5} + \frac{3}{(s+3)^2+9} \rightarrow \boxed{y(t) = e^{-5t} + \cos(3t)\,e^{-3t}}$$ ✓

g) $y''(t) - 2\,y'(t) + 5\,y(t) = -8\,e^{-t}$ $\qquad y(0) = 2 \ \wedge \ y'(0) = 12$

$$s^2\,Y(s) - s\,\overset{2}{y(0)} - \overset{12}{y'(0)} - 2\left[s\,Y(s) - \overset{2}{y(0)}\right] + 5\,Y(s) = \frac{-8}{s+1}$$

$$s^2\,Y(s) - 2s - 12 - 2s\,Y(s) + 4 + 5\,Y(s) = \frac{-8}{s+1}$$

$$Y(s)\,(s^2-2s+5) = -\frac{8}{s+1} + 2s + 8 = \frac{-8 + 2\overbrace{(s+4)(s+1)}^{s^2+5s+4}}{s+1} = \frac{2s^2+10s}{s+1}$$

$$Y(s) = \frac{2s^2+10s}{(s+1)\,(s^2-2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$\rightarrow A(s^2-2s+5) + B\,(s^2+s) + C\,(s+1) = 2s^2+10s \rightarrow \begin{cases} A + B = 2 \\ -2A + B + C = 10 \\ 5A + C = 0 \end{cases} \rightarrow \begin{matrix} A = -1 \\ B = 3 \\ C = 5 \end{matrix}$$

$$Y(s) = \frac{-1}{s+1} + \frac{3s+5}{(s-1)^2+4} = \frac{-1}{s+1} + \frac{3s-3+8}{(s-1)^2+4} = \frac{-1}{s+1} + \frac{3(s-1)}{(s-1)^2+4} + \frac{4 \cdot 2}{(s-1)^2+4}$$

$$\boxed{y(t) = -e^{-t} + 3\cos(2t)\,e^{t} + 4\,\mathrm{sen}(2t)\,e^{t}}$$ ✓

Sy1

h) $y''(t) - 3y'(t) + 2y(t) = \cos(t)$ $\qquad y(0) = 0 \wedge y'(0) = -1$

$$s^2 Y(s) - s\overbrace{y(0)}^{0} - \overbrace{y'(0)}^{-1} - 3\left[sY(s) - \overbrace{y(0)}^{0}\right] + 2Y(s) = \frac{s}{s^2+1}$$

$$Y(s)(s^2 - 3s + 2) = \frac{s}{s^2+1} - 1 = \frac{s - s^2 - 1}{s^2+1}$$

$$Y(s) = \frac{-s^2 + s - 1}{(s^2+1)(s^2-3s+2)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{Cs+D}{s^2+1} =$$

raíces compl. $\qquad$ raíces reales: 2, 1

$$= \frac{A\overbrace{(s-2)(s^2+1)}^{s^3+s-2s^2-2} + B\overbrace{(s-1)(s^2+1)}^{s^3-s^2+s-1} + C(s^3-3s^2+2s) + D(s^2-3s+2)}{(s-1)(s-2)(s^2+1)}$$

$$\begin{array}{ll} s^3: & A + B + C = 0 \\ s^2: & -2A - B - 3C + D = -1 \\ s: & A + B + 2C - 3D = 1 \\ \#: & -2A - B + 2D = -1 \end{array}$$

$\frac{-s^2+s-1}{(s-1)(s-2)(s^2+1)} \to \boxed{A = 1/2}$ $\cdot(s-1)$ $\qquad \boxed{C = 1/10}$

$\boxed{B = -3/5}$ $(s-2)$ $\qquad \boxed{D = -3/10}$

$$Y(s) = \frac{-3/2}{s-1} + \frac{7/5}{s-2} + \frac{\frac{1}{10}s - \frac{3}{10}}{s^2+1} = \frac{-3/2}{(s-1)} + \frac{7/5}{(s-2)} + \frac{\frac{1}{10}s}{s^2+1} - \frac{3/10}{s^2+1}$$

$$\boxed{y(t) = \frac{1}{2}e^{t} - \frac{3}{5}e^{2t} + \frac{1}{10}\cos(t) - \frac{3}{10}\operatorname{sen}(t)}$$

i) $y''(t) + 9y(t) = \cos(2t)$ $\qquad \boxed{y'(0) = a}$ $y(0) = 1 \wedge y\left(\frac{\pi}{2}\right) = -1$

$$s^2 Y(s) - s\overbrace{y(0)}^{1} - \overbrace{y'(0)}^{a} + 9Y(s) = \frac{s}{s^2+4} \rightarrow Y(s)(s^2+9) = \frac{s}{s^2+4} + s + a$$

$$Y(s) = \frac{\overbrace{s + (s+a)(s^2+4)}^{s + s^3 + 4s + as^2 + 4a}}{(s^2+4)(s^2+9)} = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{s^2+9} = \frac{A(s^3+9s) + B(s^2+9) + C(s^3+4s) + D(s^2+4)}{(s^2+4)(s^2+9)}$$

$$\begin{array}{ll} s^3: & A + C = 1 \\ s^2: & B + D = a \\ s: & 9A + 4C = 5 \\ \#: & 9B + 4D = 4a \end{array} \qquad \left.\begin{array}{l} A + C = 1 \\ 9A + 4C = 5 \end{array}\right\} \rightarrow \begin{array}{l} A = 1/5 \\ C = 4/5 \end{array} \qquad \boxed{Y(s) = \frac{1/5\, s}{s^2+4} + \frac{4/5\, s + a}{s^2+9}}$$

$$\left.\begin{array}{l} B + D = a \\ 9B + 4D = 4a \end{array}\right\} 9B + 4D = 4(B+D) = 4B + 4D \rightarrow 5B = 0 \rightarrow B = 0 \rightarrow D = a$$

$$Y(s) = \frac{\frac{1}{5}s}{s^2+4} + \frac{\frac{4}{5}s}{s^2+9} + \frac{3\,a}{3(s^2+9)} \longrightarrow \boxed{y(t) = \frac{1}{5}\cos(2t) + \frac{4}{5}\cos(3t) + \frac{a}{3}\operatorname{sen}(3t)}$$

$$y\left(\frac{\pi}{2}\right) = -1 = \frac{1}{5}\overbrace{\cos\left(2\frac{\pi}{2}\right)}^{-1} + \frac{4}{5}\overbrace{\cos\left(3\frac{\pi}{2}\right)}^{0} + \frac{a}{3}\overbrace{\operatorname{sen}\left(\frac{3\pi}{2}\right)}^{-1} = -\frac{1}{5} - \frac{a}{3} = -1 \rightarrow \boxed{a = \frac{12}{5}}$$

$$\boxed{y(t) = \frac{1}{5}\cos(2t) + \frac{4}{5}\cos(3t) + \frac{4}{5}\operatorname{sen}(3t)}$$

Mats Superior

T. Laplace

⑨ Resuelva los siguientes sistemas de Ecuaciones Diferenciales por medio de la transformada de Laplace

a) $\begin{cases} y'(t) + 1/2\, x(t) = 1 \\ x'(t) + 2y(t) = 2t \end{cases}$ con $x(0)=0$, $y(0)=0$

$$\begin{cases} s\,Y(s) - \overset{0}{y(0)} - \frac{1}{2}X(s) = \frac{1}{s} \\ s\,X(s) - \overset{0}{x(0)} + 2Y(s) = \frac{2}{s^2} \end{cases} \rightarrow \begin{cases} X(s)(-1/2) + Y(s)\,s = 1/s \\ X(s)\,s + Y(s)\,2 = 2/s^2 \end{cases}$$

$$\Delta = \begin{vmatrix} -1/2 & s \\ s & 2 \end{vmatrix} = -1 - s^2 = \boxed{-(s^2+1) = \Delta} \qquad X(s) = \frac{\begin{vmatrix} 1/s & s \\ 2/s^2 & 2 \end{vmatrix}}{\Delta} = \frac{\left|\frac{2}{s} - \frac{2}{s}\right|}{\Delta} = 0 \rightarrow \boxed{x(t)=0}$$

$$Y(s) = \frac{\begin{vmatrix} -1/2 & 1/s \\ s & 2/s^2 \end{vmatrix}}{\Delta} = \frac{-\frac{1}{s^2} - 1}{-(s^2+1)} = \frac{-\frac{1+s^2}{s^2}}{-(s^2+1)} = \frac{1+s^2}{s^2} \cdot \frac{1}{s^2+1} = \frac{1}{s^2} = Y(s) \rightarrow \boxed{y(t) = t}$$

b) $\begin{cases} x'(t) = x(t) - y(t) \\ y'(t) = 2x(t) + 4y(t) \end{cases}$ con $x(0) = -1$, $y(0) = 0$

$$\begin{cases} s\,X(s) - \overset{-1}{x(0)} = X(s) - Y(s) \\ s\,Y(s) - \overset{0}{y(0)} = 2X(s) + 4Y(s) \end{cases} \rightarrow \begin{cases} X(s)(s-1) + Y(s) = -1 \\ X(s)(-2) + Y(s)(s-4) = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} s-1 & 1 \\ -2 & s-4 \end{vmatrix} = (s-1)(s-4) + 2 = \boxed{s^2 - 5s + 6 = \Delta}$$

$$\begin{cases} A + B = -1 \\ -2A - 3B = 4 \end{cases} \quad A = 1,\ B = -2$$

$$X(s) = \frac{\begin{vmatrix} -1 & 1 \\ 0 & s-4 \end{vmatrix}}{\Delta} = \frac{-(s-4)}{s^2-5s+6} = \frac{A}{(s-3)} + \frac{B}{(s-2)} \rightarrow A(s-2) + B(s-3) = -s+4$$

$$X(s) = \frac{1}{s-3} - \frac{2}{s-2} \longrightarrow \boxed{x(t) = e^{3t} - 2e^{2t}}$$

$$Y(s) = \frac{\begin{vmatrix} s-1 & -1 \\ -2 & 0 \end{vmatrix}}{\Delta} = \frac{-2}{s^2-5s+6} = \frac{A(s-2) + B(s-3)}{(s-2)(s-3)} \rightarrow \begin{cases} A + B = 0 \\ -2A - 3B = -2 \end{cases} \quad A = -2,\ B = 2$$

$$Y(s) = \frac{-2}{s-3} + \frac{2}{s-2} \longrightarrow \boxed{y(t) = -2e^{3t} + 2e^{2t}}$$

Sylvime

c) $\begin{cases} x'(t) + 2y(t) = 0 \\ x'(t) - y'(t) = 0 \end{cases}$ con $x(0) = -1$; $y(0) = 2$

resto m.a.m: $2y(t) + y'(t) = 0$

$2Y(s) + sY(s) - \overset{2}{y(0)} = 0$

$Y(s)(s+2) = +2$

$Y(s) = \dfrac{+2}{s+2} \rightarrow \boxed{y(t) = +2e^{-2t}}$

$\rightarrow x'(t) + 2y(t) = x'(t) + 4e^{-2t} = 0$

$x'(t) = -4e^{-2t}$

$sX(s) - \overset{-1}{x(0)} = \dfrac{-4}{s+2} \rightarrow X(s) = \left(\dfrac{-4}{s+2} - 1\right) \cdot \dfrac{1}{s} = \dfrac{-4-(s+2)}{(s+2)s} = \dfrac{-s-6}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$

$A(s+2) + Bs = -s-6 \rightarrow \begin{cases} A + B = -1 \\ 2A = -6 \end{cases}$ $B = 2$, $\rightarrow A = -3$ $\rightarrow X(s) = \dfrac{-3}{s} + \dfrac{2}{s+2}$

$\boxed{x(t) = 3 + 2e^{-2t}}$

d) $\begin{cases} x'(t) + 2y'(t) + 5x(t) = 0 \\ y'(t) + y(t) + 2x(t) = 0 \end{cases}$ $y(0) = 1 \wedge x(0) = 0$

$\begin{cases} sX(s) - \overset{0}{x(0)} + 2[sY(s) - \overset{1}{y(0)}] + 5X(s) = 0 \\ sY(s) - \overset{1}{y(0)} + Y(s) + 2X(s) = 0 \end{cases} \rightarrow \begin{cases} X(s)(s+5) + Y(s)\,2s = 2 \\ X(s)\,2 + Y(s)(s+1) = 1 \end{cases}$

$\Delta = \begin{vmatrix} s+5 & 2s \\ 2 & s+1 \end{vmatrix} = (s+5)(s+1) - 4s = s^2 + 6s + 5 - 4s = \boxed{s^2 + 2s + 5 = \Delta}$

$X(s) = \dfrac{\begin{vmatrix} 2 & 2s \\ 1 & s+1 \end{vmatrix}}{\Delta} = \dfrac{2s+2-2s}{s^2+2s+5} = \dfrac{2}{(s+1)^2+4} \longrightarrow \boxed{x(t) = \operatorname{sen}(2t)\,e^{-t}}$

$Y(s) = \dfrac{\begin{vmatrix} s+5 & 2 \\ 2 & 1 \end{vmatrix}}{\Delta} = \dfrac{(s+5-4)}{s^2+2s+5} = \dfrac{s+1}{(s+1)^2+4} \longrightarrow \boxed{y(t) = \cos(2t)\,e^{-t}}$

Mat Sup.
T. Laplace

e) $\begin{cases} 2x'(t) - y'(t) = 4t - 3 \\ x'(t) = y(t) - t \end{cases}$ $\quad y(0) = 2 \ , \ x(0) = 1$

$$\begin{cases} 2sX(s) - 2\overset{1}{x(0)} - sY(s) + \overset{2}{y(0)} = \frac{4}{s^2} - \frac{3}{s} \\ sX(s) - \overset{1}{x(0)} = Y(s) - \frac{1}{s^2} \end{cases}$$

$$\begin{cases} X(s)\,(2s) + Y(s)\,(-s) = \frac{4-3s}{s^2} \\ X(s)\,s + Y(s)\,(-1) = \frac{s^2-1}{s^2} \end{cases} \qquad \Delta = \begin{vmatrix} 2s & -s \\ s & -1 \end{vmatrix} = \boxed{-2s + s^2 = \Delta}$$

$$X(s) = \frac{\begin{vmatrix} \frac{4-3s}{s^2} & -s \\ \frac{s^2-1}{s^2} & -1 \end{vmatrix}}{\Delta} = \frac{\frac{3s-4}{s^2} + \frac{s^2-1}{s}}{s^2 - 2s} = \frac{\frac{3s-4+s^3-s}{s^2}}{s^2-2s} = \frac{s^3+2s-4}{s^2} \cdot \frac{1}{s(s-2)} =$$

$$= \frac{s^3+2s-4}{s^3\,(s-2)} = \frac{A}{s^3} + \frac{B}{s-2} = \frac{A(s-2) + Bs^3}{s^3\,(s-2)} \rightarrow \begin{cases} B = 1 \\ A = 2 \\ -2A = -4 \end{cases}$$

$$X(s) = \frac{2}{s^3} + \frac{1}{s-2} \longrightarrow \boxed{x(t) = t^2 + e^{2t}}$$

$$Y(s) = \frac{\begin{vmatrix} 2s & \frac{4-3s}{s^2} \\ s & \frac{s^2-1}{s^2} \end{vmatrix}}{\Delta} = \frac{\frac{2s^2-2-4+3s}{s}}{s^2-2s} = \frac{2s^2+3s-6}{s^2\,(s-2)} = \frac{A(s-2) + Bs^2}{s^2(s-2)}$$

$$\begin{cases} B = 2 \\ A = 3 \\ -2A = -6 \end{cases} \rightarrow Y(s) = \frac{3}{s^2} + \frac{2}{s-2} \rightarrow \boxed{y(t) = 3t + 2e^{2t}}$$

Sul

PARTE 4: Convolución

(10) Halle $\mathcal{L}\left[\int_0^t \underbrace{u\cos(u)}_{f(u)}\ \underbrace{e^{(t-u)}}_{g(t-u)}\,du\right]$ utilizando el teorema de Convolución.

- $f(u) = u^1 \underbrace{\cos(u)}_{h(u)} \rightarrow F(s) = (-1)^1\ H'(s)$ $\boxed{= \dfrac{s^2-1}{(s^2+1)^2} = F(s)}$

$\hookrightarrow H(s) = \dfrac{s}{s^2+1} \longrightarrow H'(s) = \dfrac{1-s^2}{(s^2+1)^2}$

- $g(t) = e^t \rightarrow \boxed{G(s) = \dfrac{1}{s-1}}$

$\mathcal{L}\left[\int_0^t f(u)\ g(t-u)\,du\right] = F(s)\ G(s)$

$\rightarrow \mathcal{L}\left[\int_0^t u\cos(u).e^{(t-u)}\,du\right] = \dfrac{s^2-1}{(s^2+1)^2} \cdot \dfrac{1}{(s-1)} = \dfrac{(s+1)\cancel{(s-1)}}{(s^2+1)^2\cancel{(s-1)}}$

$$\boxed{\mathcal{L}\left[\int_0^t u\cos(u)\ e^{t-u}\,du\right] = \frac{s+1}{(s^2+1)^2}}$$

(11) Antitransforme utilizando Convolución:

a) $Y(s) = \dfrac{6}{(s+1)(s^2+4)} = \dfrac{3}{s+1} \cdot \dfrac{2}{s^2+4} \rightarrow F(s) = \dfrac{3}{s+1} \rightarrow f(t) = 3e^{-t}$

$G(s) = \dfrac{2}{s^2+4} \rightarrow g(t) = \operatorname{sen}(2t)$

$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[F(s)\ G(s)] = \int_0^t \operatorname{sen}(2u).3e^{-(t-u)}\,du =$

$= \int_0^t \operatorname{sen}(2u)\,3e^{-t}\,e^{u}\,du = 3e^{-t}.\ \dfrac{e^u(\operatorname{sen}(2u) - 2\cos(2u))}{1^2+2^2}\Big|_0^t =$

$= \dfrac{3e^{-t}}{5}\left[e^t(\operatorname{sen}(2t) - 2\cos(2t)) - \overset{1}{e^0}(\overset{0}{\operatorname{sen}(2.0)} + \overset{2}{2\cos(2.0)})\right] =$

$= \dfrac{3}{5}e^{-t}\left[e^t(\operatorname{sen}(2t) - 2\cos(2t)) - 2\right]$

$$\boxed{y(t) = \frac{3}{5}\operatorname{sen}(2t) - \frac{6}{5}\cos(2t) - \frac{6}{5}e^{-t}}$$

$2\,\text{sen}(x)\,\text{sen}(y) = \text{sen}(x-y) + \text{sen}(x+y)$

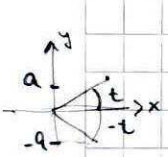
Mat Sup.
T. Laplace

b) $Y(s) = \dfrac{1}{(s^2+1)^2} = \dfrac{1}{s^2+1} \cdot \dfrac{1}{s^2+1} \rightarrow \boxed{F(s) = G(s) = \dfrac{1}{s^2+1}}$

$f(t) = \text{sen}(t) \qquad g(t) = \text{sen}(t)$

$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[F(s) \cdot G(s)] = \displaystyle\int_0^t \text{sen}(\overset{x}{u})\,\text{sen}(\overset{y}{t-u})\,du \overset{\text{Prop }s}{=}$

$= \dfrac{1}{2}\displaystyle\int_0^t \cos(2u-t) - \cos(t)\,du = \dfrac{1}{2}\left[\int_0^t \cos(2u-t)\,dt - \cos(t)\int_0^t du\right] =$



$= \dfrac{1}{2}\left[\dfrac{\text{sen}(2u-t)}{2}\Big|_0^t - \cos(t)\,t\right] = \dfrac{1}{2}\left[\dfrac{\text{sen}(t) - \overbrace{\text{sen}(-t)}^{-\text{sen}(t)}}{2} - t\cos(t)\right] =$

$= \boxed{\dfrac{1}{2}\left[\text{sen}(t) - t\cos(t)\right] = y(t)}$ ✓

c) $Y(s) = \dfrac{s^2}{(s^2+1)^2}$

$Y(s) = \dfrac{s}{s^2+1} \cdot \dfrac{s}{s^2+1} = F(s)\,G(s) \longrightarrow f(t) = g(t) = \cos(t)$

$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[F(s)\,G(s)] = \displaystyle\int_0^t \cos(\overset{x}{u})\cos(\overset{y}{t-u})\,du =$

$= \dfrac{1}{2}\displaystyle\int_0^t \cos(2u-t) + \cos(t)\,du = \dfrac{1}{2}\left[\int_0^t \cos(2u-t)\,du + \cos(t)\cdot t\right] =$

$= \dfrac{1}{2}\left[\dfrac{\text{sen}(2u-t)}{2}\Big|_0^t + \cos(t)\,t\right] = \dfrac{1}{2}\left[\dfrac{\text{sen}(t) - \overbrace{\text{sen}(-t)}^{-\text{sen}(t)}}{2} + t\cos(t)\right] =$

$= \dfrac{1}{2}\left[\dfrac{2\,\text{sen}(t)}{2} + t\cos(t)\right] = \boxed{\dfrac{1}{2}(\text{sen}(t) + \cos(t)) = y(t)}$ ✓

Sgl

Mat Sup.
T. Laplace

(12) Resuelva las sig. ecuaciones integrales e integrodiferenciales por medio de la transformada de Laplace, utilizando el teorema de convolución:

a) $y(t) + 4\int_0^t \underbrace{(t-u)^2}_{g(t-u)} \underbrace{y(u)}_{F(u)}\, du = t^2$

$G(s) = \dfrac{2}{s^3}$ $\quad F(s) = Y(s)$

$$Y(s) + 4\,F(s)\,G(s) = \frac{2}{s^3}$$

$$Y(s) + 4\,Y(s)\cdot\frac{2}{s^3} = \frac{2}{s^3}$$

$$Y(s)\left(1+\frac{8}{s^3}\right) = \frac{2}{s^3} = Y(s)\left(\frac{s^3+8}{s^3}\right)$$

$$\rightarrow Y(s) = \frac{2}{\cancel{s^3}}\,\frac{\cancel{s^3}}{s^3+8} = \boxed{\frac{2}{s^3+8} = Y(s)}$$

1 raíz de s^3+8 es real: -2

	s^3	s^2	s	#
	1	0	0	8
-2		-2	4	-8
	1	-2	4	0

$$Y(s) = \frac{2}{(s+2)(s^2-2s+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2-2s+4}$$

$\rightarrow A(s^2-2s+4) + B(s^2+2s) + C(s+2) = 2$

$$\begin{cases} A + B = 0 \\ -2A + 2B + C = 0 \\ 4A + 2C = 2 \end{cases} \rightarrow \begin{matrix} A = 1/6 \\ B = -1/6 \\ C = 2/3 \end{matrix} \rightarrow Y(s) = \frac{1/6}{s+2} + \frac{-1/6\, s + 2/3}{(s-1)^2+3}$$

$$Y(s) = \frac{1/6}{s+2} + \frac{-1/6\, s + \overbrace{1/6 + 1/2}^{2/3}}{(s-1)^2+3} = \frac{1/6}{s+2} + \frac{-1/6(s-1)}{(s-1)^2+3} + \frac{\sqrt{3}}{\sqrt{3}}\frac{1/2}{(s-1)^2+3} = Y(s)$$

$$\rightarrow \boxed{y(t) = \frac{1}{6}e^{-2t} - \frac{1}{6}\cos(\sqrt{3}t)\,e^{t} + \frac{1}{2\sqrt{3}}\,\mathrm{sen}(\sqrt{3}t)\,e^{t}}$$

Sylvina

b) $y(t) = t + 2\int_0^t \underbrace{\cos(t-u)}_{g(t-u)}\ \underbrace{y(u)}_{f(u)}\,du \rightarrow F(s) = Y(s)$

$G(s) = \frac{s}{s^2+1}$

$$Y(s) = \frac{1}{s^2} + 2\cdot F(s)\cdot G(s) = \frac{1}{s^2} + 2\,Y(s)\cdot\frac{s}{s^2+1}$$

$$Y(s)\left(1 - \frac{2s}{s^2+1}\right) = Y(s)\left(\frac{s^2+1-2s}{s^2+1}\right) = \frac{1}{s^2} \rightarrow Y(s) = \frac{s^2+1}{s^2(\underbrace{s^2-2s+1}_{\text{raíces reales } (s-1)^2})}$$

$$Y(s) = \frac{As+B}{s^2} + \frac{Cs+D}{(s^2-2s+1)}$$

$$A\overbrace{s(s^2-2s+1)}^{s^3-2s^2+s} + B(s^2-2s+1) + Cs^3 + Ds^2 = s^2+1 \rightarrow \begin{cases} A+C = 0 \\ -2A+B+D = 1 \\ A-2B = 0 \quad \boxed{A=2} \\ B = 1 \end{cases}$$

$\boxed{B=1} \rightarrow \boxed{A=2} \rightarrow \boxed{C=-2} \rightarrow \boxed{D=4}$

$$Y(s) = \frac{2s+1}{s^2} + \frac{-2s+4}{(s-1)^2} = \frac{2}{s} + \frac{1}{s^2} + \frac{-2s+2+2}{(s-1)^2} =$$

$$= \frac{2}{s} + \frac{1}{s^2} - \frac{2\cancel{(s-1)}}{(s-1)^{\cancel{2}}} + \frac{2}{(s-1)^2} = \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s-1} + 2\cdot\frac{1}{(s-1)^2}$$

$$\boxed{y(t) = 2 + t - 2e^t + 2te^t}$$

c) $\int_0^t \underbrace{y(u)}_{f(u)}\ \underbrace{(t-u)}_{g(t-u)}\,du = \frac{t}{2} - \frac{\operatorname{sen}(2t)}{4}$

$f(u) = y(u) \rightarrow F(s) = Y(s)$

$g(t) = t \rightarrow G(s) = \frac{1}{s^2}$

• $Y(s)\,\frac{1}{s^2} = \frac{1}{2s^2} - \frac{1}{\cancel{4}_{2}}\cdot\frac{\cancel{2}^{1}}{s^2+4} = \frac{1}{2s^2} - \frac{1}{2}\cdot\frac{1}{s^2+4}$

$$Y(s) = \frac{\cancel{s^2}}{2\cancel{s^2}} - \frac{s^2}{2(s^2+4)} = \frac{1}{2} - \frac{1}{2}\,\frac{s^2}{(s^2+4)} = \frac{s^2+4-s^2}{2(s^2+4)} = \frac{4}{2(s^2+4)} = \frac{2}{s^2+4}$$

$$Y(s) = \frac{2}{s^2+4} \rightarrow \boxed{y(t) = \operatorname{sen}(2t)}$$

d) $y(t) = \int_0^t y(u)\,du + \cos(t)$

$$Y(s) = \frac{Y(s)}{s} + \frac{s}{s^2+1} \rightarrow Y(s)\left(1 - \frac{1}{s}\right) = Y(s)\left(\frac{s-1}{s}\right) = \frac{s}{s^2+1} \rightarrow Y(s) = \frac{s^2}{(s-1)(s^2+1)}$$

$$= \frac{A}{(s-1)} + \frac{Bs+C}{(s^2+1)} = \frac{A(s^2+1) + B(s^2-s) + C(s-1)}{(s-1)(s^2+1)} \rightarrow \begin{cases} A+B = 1 \\ -B+C = 0 \\ A-C = 0 \end{cases} \rightarrow \begin{matrix} A = 1/2 \\ B = 1/2 \\ C = 1/2 \end{matrix}$$

$$Y(s) = \frac{1/2}{s-1} + \frac{1/2\, s}{s^2+1} + \frac{1/2}{s^2+1} \rightarrow \boxed{y(t) = \frac{1}{2}e^t + \frac{1}{2}\cos(t) + \frac{1}{2}\operatorname{sen}(t)}$$

Mat Sup.
T. Laplace

e) $\int_0^t \underbrace{y(u)}_{f(u)}\,\underbrace{(t-u)}_{g(t-u)}\,du + y'(t) = \frac{t^4}{12} + t + 1 \qquad$ con $y(0) = -1$

$g(t-u) \to g(t) = t$

$$Y(s)\cdot\frac{1}{s^2} + sY(s) - \overset{-1}{y(0)} = \frac{4!}{12\,s^5} + \frac{1}{s^2} + \frac{1}{s}$$

$$Y(s)\underbrace{\left(\frac{1}{s^2}+s\right)}_{\frac{1+s^3}{s^2}} = \frac{2}{s^5} + \frac{1}{s^2} + \frac{1}{s} - 1 = \frac{2+s^3+s^4-s^5}{s^5}$$

$$Y(s) = \frac{-s^5+s^4+s^3+2}{\cancel{s^5}\,s^3}\cdot\frac{\cancel{s^2}}{1+s^3} = \frac{-s^5+s^4+s^3+2}{s^3+s^6} = \frac{-s^5+s^4+s^3+2}{s^3(1+s^3)} =$$

$$= \frac{-s^5+s^4+s^3}{s^3(1+s^3)} + \frac{2}{s^3(1+s^3)} = \frac{\cancel{s^3}(-s^2+s+1)}{\cancel{s^3}(1+s^3)} + \frac{2}{s^3(1+s^3)}$$

$$Y(s) = \frac{-s^5+s^4+s^3+2}{s^3(1+s^3)} = \frac{\overset{=2}{A}}{s^3} + \frac{Bs^2+Cs+D}{1+s^3} = \frac{2}{s^3} + \frac{-s^2+s-1}{1+s^3} = \frac{2}{s^3} + \frac{\overset{-1}{\cancel{(-s^2+s-1)}}}{(s+1)\cancel{(s^2-s+1)}}$$

$Bs^5 + Cs^4 + Ds^3 + 2 + 2s^3 \to B = -1$
$C = 1$
$D+2 = 1 \to D = -1$

$$\to Y(s) = \frac{2}{s^3} - \frac{1}{s+1} \to \boxed{y(t) = t^2 - e^{-t}}$$

f) $y'(t) = 1 - \int_0^t \underbrace{y(u)}_{f(u)}\,\underbrace{e^{2(t-u)}}_{g(t-u)}\,du \qquad y(0) = 1$

$g(t-u) \to g(t) = e^{2t} \to G(s) = \frac{1}{s-2}$

$$sY(s) - \overset{1}{y(0)} = \frac{1}{s} - Y(s)\cdot\frac{1}{s-2}$$

$$Y(s)\left(s + \frac{1}{s-2}\right) = Y(s)\left(\frac{s^2-2s+1}{s-2}\right) = \frac{1}{s} + 1 = \frac{1+s}{s}$$

$$Y(s) = \overbrace{\frac{(s+1)}{s}\cdot\frac{(s-2)}{(s-1)^2}}^{s^2-s-2} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{(s-1)^2} = \frac{A(s^2-2s+1)+B(s^2-s)+Cs}{s(s-1)^2}$$

$$\to \begin{cases} A+B = 1 \\ -2A-B+C = -1 \\ A = -2 \end{cases} \to \begin{matrix} A = -2 \\ B = 3 \\ C = -2 \end{matrix} \to Y(s) = \frac{-2}{s} + \frac{3}{s-1} - \frac{2}{(s-1)^2}$$

$$\boxed{y(t) = -2 + 3e^t - 2te^t}$$

$$\mathcal{L}[f(t)] = \int_0^\infty f(t) \cdot e^{-st}\, dt = F(s)$$

PARTE 5: Evaluaciones integrales

(13) Calcule el valor de los sig. integrales utilizando transformadas de Laplace:

a) $\int_0^\infty t^3 e^{-t} \operatorname{sen}(t)\, dt$ $\leftarrow$ $\boxed{s=1}$

$$\mathcal{L}[\underbrace{t^3 \operatorname{sen}(t)}_{f(t)}] = \int_0^\infty t^3 \operatorname{sen}(t)\, e^{-st}\, dt = F(s)$$

$$\mathcal{L}[t^3 \underbrace{\operatorname{sen}(t)}_{g(t)}] = (-1)^3\, G'''(s)$$

$$\to G(s) = \frac{1}{s^2+1} = (s^2+1)^{-1}$$

$$\to G'(s) = \frac{-2s}{(s^2+1)^2} = -2s(s^2+1)^{-2} \longrightarrow G''(s) = -2(s^2+1)^{-2} + \overbrace{(-2s)(-2(s^2+1)^{-3} \cdot 2s)}^{+8s^2(s^2+1)^{-3}}$$

$$\to G'''(s) = 4(s^2+1)^{-3} \cdot 2s + 8\left[2s(s^2+1)^{-3} + s^2(-3(s^2+1)^{-4}\, 2s)\right]$$

$$G'''(s) = 8s(s^2+1)^{-3} + 16s(s^2+1)^{-3} - 48s^3(s^2+1)^{-4} \to \boxed{F(1) = -G'''(1) = 0}\ ✓$$

b) $\int_0^\infty e^{-3t}\, t \operatorname{sen}(t)\, dt = \frac{3}{50}$ ✓ $\quad (s=3)$

$$f(t) = t \underbrace{\operatorname{sen}(t)}_{g(t)} \longrightarrow F(s) = (-1)^1 \cdot G'(s) = \frac{2s}{(s^2+1)^2} = F(s)$$

$$G(s) = \frac{1}{s^2+1} \longrightarrow G'(s) = \frac{-2s}{(s^2+1)^2}$$

$$F(3) = \frac{6}{100} = \frac{3}{50}$$

c) $\int_0^\infty \underbrace{t^2}_{f(t)} e^{-2t}\, dt = \frac{1}{4}$ ✓ $\quad s=2$

$$\to F(s) = \frac{2}{s^3} \longrightarrow F(2) = \frac{2}{8} = \frac{1}{4}$$

d) $\int_0^\infty \frac{e^{-3t} - e^{-6t}}{t}\, dt = \int_0^\infty e^{-3t}\left(\frac{\overbrace{1-e^{-3t}}^{f(t)}}{t}\right) dt \overset{s=3}{=} F(3)$ $\quad \leftarrow \frac{f(t)}{t}$

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^\infty F(u)\, du = \int_s^\infty \frac{1}{u} - \frac{1}{u+3}\, du = \left[\ln(u) - \ln(u+3)\right]\Big|_s^\infty =$$

$$= \ln\left(\frac{u}{u+3}\right)\Big|_s^\infty = 0 - \ln\left(\frac{s}{s+3}\right) = \ln\left(\frac{s+3}{s}\right) = F(s) \to \boxed{F(3) = \ln(2)}$$

Mat Sup.
Laplace

e) $\int_0^{\infty} t\, e^{-2t} \cos 4t\, dt \quad \boxed{s=2}$

$f(t) = t \underbrace{\cos(4t)}_{g(t)} \to F(s) = (-1)\, G'(s)$

$G(s) = \frac{s}{s^2+16} = s\,(s^2+16)^{-1} \to G'(s) = (s^2+16)^{-1} + s\,(-1)\,(s^2+16)^{-2}\, 2s =$

$= \frac{1}{s^2+16} - \frac{2s^2}{(s^2+16)^2}$

$F(s) = \frac{2s^2}{(s^2+16)^2} - \frac{1}{s^2+16} = \frac{2s^2 - s^2 - 16}{(s^2+16)^2} = \frac{s^2-16}{(s^2+16)^2} \to \boxed{F(2) = -0{,}03}$ ✓

EJERCICIOS PARA MARCAR LA RESPUESTA CORRECTA, tomado en finales:

(14) Marcar la única respuesta correcta de cada ítem:

(1) Sabiendo que $g(t) = e^{5t} f(t)$ entonces:

✓ a) $\lim_{s\to\infty} s\,F(s) = \lim_{s\to\infty} s\,G(s)$

b) $\mathcal{L}[f(t)] = G(s-5)$

c) $\mathcal{L}[g(t)] = F(s) \cdot \frac{1}{s-5}$

d) ninguna es correcta

x T. V. inicial

$\lim_{s\to\infty} s\,G(s) = \lim_{t\to 0} g(t) = \lim_{t\to 0} e^{5t} f(t) = \lim_{t\to 0} \overset{\to 1}{e^{5t}} \cdot \lim_{t\to 0} f(t) = 1 \lim_{s\to\infty} s\,F(s)$

(2) Sabiendo que $F(s) = \mathcal{L}[f(t)] \wedge G(s) = \mathcal{L}[g(t)]$ entonces:

~~a)~~ $\mathcal{L}[f(t) \cdot g(t)] = F(s) \cdot G(s)$

✓ b) Si $f(t) = \delta(t)$ entonces $F(s) \cdot G(s) = G(s)$ $\quad f(t) = \delta(t) \to F(s) = 1$

~~c)~~ Si $f(t) = 1$ entonces $F(s) \cdot G(s) = G(s)$

d) ninguna de las anteriores

(3) Sea $f(t) = t$ si $t \in [0,1]$ y $f(t) = 0$ si $t \notin [0,1]$ entonces:

a) No existe transformada de Laplace de $f(t)$

b) $\mathcal{L}[f(t)] = \frac{1}{s^2}$

✓ c) $\mathcal{L}[f(t)] = \frac{1}{s^2}(1-e^{-s}) - \frac{e^{-s}}{s}$ $\quad$ ver justificación después del ej. 11

d) $\mathcal{L}[f(t)] = \frac{1}{s^2}(1-e^{-s})$

Sgl

(4) La solución de la ecuación $y''(t) - 2y'(t) + 5y(t) = 3e^t \operatorname{sen}(t)$ con $y(0) = 0$, $y'(0) = 1$ es:

a) $y(t) = e^{-2t}[\cos(t) + \operatorname{sen}(t)]$

b) $y(t) = e^{2t} \operatorname{sen}(t)$

/ c) $y(t) = e^t \operatorname{sen}(t)$

d) ninguna de los anteriores

$$s^2 Y(s) - s\overset{0}{y(0)} - \overset{1}{y'(0)} - 2[sY(s) - \overset{0}{y(0)}] + 5Y(s) = \frac{3}{(s-1)^2+1}$$

$$Y(s)(s^2 - 2s + 5) = \frac{3}{(s-1)^2+1} + 1 = \frac{3 + (s-1)^2 + 1}{(s-1)^2+1} = \frac{s^2 - 2s + 1 + 4}{(s-1)^2+1} = \frac{s^2-2s+5}{s^2-2s+2}$$

$$Y(s) = \frac{\cancel{(s^2-2s+5)}}{(s^2-2s+2)} \frac{1}{\cancel{(s^2-2s+5)}} = \frac{1}{(s-1)^2+1} \to y(t) = e^t \operatorname{sen}(t)$$

(5) La antitransformada de Laplace de $Y(s) = 2(s^2+1)^{-2}$ es:

/ a) $y(t) = 2\operatorname{sen}^2(t)$ b) $y(t) = \operatorname{sen}(t) - t\cos(t)$ c) $y(t) = 2t\operatorname{sen}(t)$ d) ninguna de ant.

Pruebo con b: $y(t) = \operatorname{sen}(t) - t\overbrace{\cos(t)}^{f(t)}$

$$f(t) = \cos(t) \to F(s) = \frac{s}{s^2+1} \to \mathcal{L}[t\cos(t)] = (-1)F'(s) = \frac{s^2-1}{(s^2+1)^2}$$

$$F(s) = s(s^2+1)^{-1} \to F'(s) = \frac{1}{s^2+1} + s(-1)(s^2+1)^{-2}(2s) = \frac{1}{s^2+1} - \frac{2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

$$y(t) = \operatorname{sen}(t) - t\cos(t) \to Y(s) = \frac{1}{s^2+1} - \frac{s^2-1}{(s^2+1)^2} = \frac{\cancel{s^2}+1-\cancel{s^2}+1}{(s^2+1)^2} = \frac{2}{(s^2+1)^2}$$

(6) El valor de la integral $\int_0^\infty \frac{e^{-2t} - e^{-6t}}{t}\,dt$, es:

/ a) 0 b) $\ln(2)$ c) $\ln(3)$ d) ninguna de los anteriores..

$$\boxed{\frac{e^{-2t} - e^{-6t}}{t} = \frac{e^{-2t}(1 - e^{-4t})}{t}} \quad F(s) = \int_0^\infty e^{-st} \cdot f(t)\,dt \to \text{tengo que hallar } F(2)$$

($s=2$)

$$y(t) = 1 - e^{-4t} \to f(t) = \frac{y(t)}{t} \to F(s) = \int_s^\infty Y(u)\,du = \int_s^\infty \frac{1}{u} - \frac{1}{u+4}\,du =$$

$$= \ln(u) - \ln(u+4)\Big|_s^\infty = \ln\left(\frac{u}{u+4}\right)\Big|_s^\infty = 0 - \ln\left(\frac{s}{s+4}\right) = \ln\left(\frac{s+4}{s}\right) \to \boxed{F(2) = \ln(3)}$$

Mat Sup
T. Laplace

(7) La transformada de Laplace de $f(t) = 3\,\delta(t) + t$ cuando $s \to \infty$, tiende a:

/ a) cero (b) tres c) infinito d) ninguna de los ant.

$$f(t) = 3\,\delta(t) + t \to F(s) = 3 + \frac{1}{s^2} \Rightarrow \lim_{s\to\infty} 3 + \underbrace{\frac{1}{s^2}}_{\to 0} = 3$$

(8) Sea $F(s) = \mathcal{L}[f(t)]$ tal que $F(s) = \frac{3s^2+9}{2s^3-23s^2+43s-42}$, el valor de $f(0)$ es:

/ a) 0 (b) 3/2 c) 3 d) infinito

$$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} s\,F(s) = \lim_{s\to\infty} \frac{3s^3+9}{2s^3-23s^2+43s-42} = \frac{3}{2}$$

(9) El valor de la integral $\int_0^\infty sen(2t)\, t^2 e^{-3t}\, dt$ es:

/ a) 0 b) $\frac{2}{13}$ (c) $\frac{92}{2197}$ d) ninguna de los ant.

tengo que hallar el valor de $F(3)$

$$F(s) = \int_0^\infty \underbrace{sen(2t)\, t^2}_{f(t)}\, e^{-st}\, dt \quad \to s = 3 \qquad f(t) = g(t)\, t^2 \to F(s) = \overbrace{(-1)^2}^{1}\, G''(s)$$

$$g(t) = sen(2t) \to G(s) = \frac{2}{s^2+4} = 2(s^2+4)^{-1} \to G'(s) = -4s(s^2+4)^{-2}$$

$$\to G''(s) = -4(s^2+4)^{-2} + (-4s)(-2)(s^2+4)^{-3}\, 2s = F(s) \to \boxed{F(3) = \frac{92}{2197}}$$

(10) La transformada de Laplace de $f(t) = -5e^{2t} + 2\cos(t)\, e^{3t} - 19\, sen(t)\, e^{3t}$ es:

/ (a) $\frac{-3s^2+s}{(s-2)(s^2-6s+10)}$

b) $\frac{s^2-4s-5}{s^3-7s^2+17s+25}$

c) $\frac{6s+3}{s^3+4s+13s}$

d) ninguna de los anteriores

$$F(s) = \frac{-5}{s-2} + \frac{2(s-3)}{(s-3)^2+1} - \frac{19}{(s-3)^2+1} = \frac{-5(s^2-6s+10) + (s-2)\overbrace{[2(s-3)-19]}^{2s-25}}{(s-2)(s^2-6s+10)} =$$

$$= \frac{-5s^2+30s-\cancel{50}+2s^2-25s-4s+\cancel{50}}{s^3-8s^2+22s-20} = \frac{-3s^2+s}{(s-2)(s^2-6s+10)}$$

Sylvina

(11) El valor de la integral : $\int_0^\infty e^{-t} \frac{\operatorname{sen}(t)}{t}\, dt$ es : $s=1$

✓ a) 0   b) 1/2   c) $\pi/4$ d) infinito

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$g(t) = \operatorname{sen}(t) \rightarrow f(t) = \frac{g(t)}{t} \rightarrow \mathcal{L}[f(t)] = \mathcal{L}\left[\frac{g(t)}{t}\right] = \int_s^\infty G(u)\, du =$$

$$\hookrightarrow G(u) = \frac{1}{u^2+1}$$

$$= \int_s^\infty \frac{1}{u^2+1}\, du = \operatorname{arctg}(u)\Big|_s^\infty = \frac{\pi}{2} - \operatorname{arctg}(s)$$

$$F(s) = \frac{\pi}{2} - \operatorname{arctg}(s) \longrightarrow F(1) = \frac{\pi}{2} - \operatorname{arctg}(1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Justificación del punto 3:

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$f(t) = \begin{cases} t & \text{si } t \in [0,1] \\ 0 & \text{si } t \notin [0,1] \end{cases} \rightarrow \mathcal{L}[f(t)] = \int_0^1 t\, e^{-st}\, dt = \frac{e^{-st}}{-s}\left(t - \frac{1}{-s}\right)\Big|_0^1 =$$

$$= -\frac{e^{-s}}{s}\left(1 + \frac{1}{s}\right) - \left[-\frac{e^0}{s}\left(0 + \frac{1}{s}\right)\right] =$$

$$= -\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{1}{s}\cdot\frac{1}{s} = \frac{1}{s^2}\left(1 - e^{-s}\right) - \frac{e^{-s}}{s}$$

RESPUESTAS EJERCICIOS DE TRANSFORMADA DE LAPLACE:

Ej. 1:

a) $Y(s) = \frac{3}{s} + \frac{5}{s^2} - \frac{4}{s^3} + \frac{6}{s^4}$; b) $Y(s) = \frac{1}{s\,(s-4)}$; c) $Y(s) = \frac{1}{s^2\,(s-a)}$

d) $Y(s) = \frac{4s}{s^2+9} - \frac{10}{s^2+4}$

Ej. 2:

a) $F(s) = \frac{2}{(s-4)^3}$; b) $F(s) = 5 \bullet \frac{s+5}{s^2+10s+34}$; c) $F(s) = \frac{4s+4}{((s+1)^2+4)^2}$

d) $F(s) = \frac{24}{s^5} \bullet e^{-s}$; e) $F(s) = \frac{2}{(s+2)^2+4} + \frac{2}{(s-3)^3}$; f) $F(s) = \frac{4}{(s+1)^3} - \frac{1}{s^2} + \frac{s}{s^2+16}$

Ej. 3:

a) $Y(s) = \frac{1-e^{-as}}{a^2 s}$ b) $Y(s) = \frac{s\sqrt{3}+5}{2(s^2+25)}$ c) $Y(s) = \frac{1}{4}\frac{s}{s^2+9} + \frac{3}{4}\frac{s}{s^2+1}$

d) $Y(s) = \frac{1}{s-\ln(a)}$ e) $Y(s) = \frac{24}{s^5} - \frac{24}{s^4} + \frac{12}{s^3} - \frac{4}{s^2} + \frac{1}{s}$ f) $Y(s) = \frac{5s^2+105}{(s^2+49)(s^2+9)}$

Ej. 4: $k = 8$

Ej. 5:

a) $y(t) = \frac{25}{2}\,t^2$ b) $y(t) = \frac{8}{\sqrt{3}}\operatorname{sen}(\sqrt{3}\,t) + 1$ c) $y(t) = 12\,e^{3t} - 8\cos(2t)$

d) $y(t) = -\,e^{-3t}$ e) $y(t) = t^3 e^t$ f) $y(t) = \cos(2t)\,e^{-3t}$

g) $y(t) = \cos(t)\,e^{-t} - 2\operatorname{sen}(t)\,e^{-t}$ h) $y(t) = t\operatorname{sen}(2t)$

Ej. 6:

a) $f(t) = -2\,e^t + 3\,e^{5t}$ b) $f(t) = \delta(t) + e^{-t} - 2\,e^{-2t}$

c) $f(t) = -e^{-2t} - t\,e^{-2t} + e^{-t}$ d) $f(t) = \frac{11}{3}\,e^{-t} - \frac{5}{2}\,e^{-2t} - \frac{1}{6}\,e^{-4t}$

e) $f(t) = \frac{1}{\sqrt{6}}\,e^{-(3-\sqrt{6})(t-0.5)} - \frac{1}{\sqrt{6}}\,e^{-(3+\sqrt{6})(t-0.5)}$ f) $f(t) = 2 - e^t + 5\,e^{6t}$

g) $f(t) = \frac{1}{3}\,e^{-t} - \frac{1}{2}\,e^{-2t} + \frac{1}{6}\,e^{2t}$ h) $f(t) = 5\,e^t + 2\,e^{2t}\cos(3t) - 5\,e^{2t}\operatorname{sen}(3t)$

i) $f(t) = -3\,e^{-t} + e^t\cos(2t) + e^t\operatorname{sen}(2t)$ j) $f(t) = 2\,e^{t-2} + 2\,e^{-2(t-2)}$

Ej. 7:

a) $\lim_{t \to \infty} f(t) = 10$ b) $\lim_{t \to 0} t\,e^{-2t} = 0$

c) i) $f(0) = 2$ ii) $\lim_{t \to \infty} f(t) = \frac{1}{2}$ iii) $f(0.5) = \frac{1}{2} + \frac{3}{2}e^{-1}$

Ej. 8:

a) $x(t) = 2 - \frac{12}{5}e^{-0.5t} + \frac{2}{5}e^{-3t}$

b) $x(t) = \frac{1}{3} - \frac{1}{3}\cos(\frac{\sqrt{15}}{2}t)\,e^{-3/2\,t} - \frac{1}{\sqrt{15}}\,\text{sen}(\frac{\sqrt{15}}{2}t)\,e^{-3/2\,t}$

c) $y(t) = \frac{6}{5} - \frac{6}{5}\,e^{-5/3\,t}$

d) $x(t) = -e^{-2t} + e^{-t}$

e) $y(t) = \frac{-13}{9}\,e^{-5t} + \frac{13}{9}\,e^{-2t} - \frac{13}{3}\,t\,e^{-2t}$

f) $y(t) = e^{-5t} + \text{sen}(3t)\,e^{-3t}$

g) $y(t) = -e^{-t} + 3\cos(2t)\,e^{t} + 4\,\text{sen}(2t)\,e^{t}$

h) $y(t) = \frac{1}{10}\cos(t) - \frac{3}{10}\,\text{sen}(t) + \frac{1}{2}\,e^{t} - \frac{3}{5}\,e^{2t}$

i) $y(t) = \frac{1}{5}\cos(2t) + \frac{4}{5}\cos(3t) + \frac{4}{5}\,\text{sen}(3t)$

OPTATIVO:

$$y(t) = -\frac{4}{25}e^{-t}\ \frac{1}{8}\,e^{-2t} + \frac{7}{200}\cos(2t) + \frac{3}{25}\,\text{sen}(2t) - \frac{1}{20}t\,\text{sen}(2t) - \frac{3}{20}t\cos(2t)$$

Ej. 9:

a) $x(t)=0$, $y(t)=t$

b) $x(t) = e^{3t} - 2\,e^{2t}$, $y(t) = -2\,e^{3t} + 2\,e^{2t}$

c) $x(t) = 2\,e^{-2t} - 3$, $y(t) = 2\,e^{-2t}$

d) $x(t) = \text{sen}(2t)\,e^{-t}$ $y(t) = \cos(2t)\,e^{-t}$

e) $x(t) = e^{2t} + t^2$, $y(t) = 2\,e^{2t} + 3\,t$

OPTATIVO:

$$y(t) = 2 + \frac{1}{2}t^2 + \frac{1}{2}e^{-t} - \frac{3}{2}sen(t) + \frac{1}{2}\cos(t), \quad z(t) = 1 - \frac{1}{2}e^{-t} + \frac{3}{2}sen(t) - \frac{1}{2}\cos(t)$$

Ej. 10:

$$Y(s) = \frac{s+1}{(s^2+1)^2}$$

Ej. 11:

a) $y(t) = \frac{3}{5}\,\text{sen}(2t) - \frac{6}{5}\cos(2t) + \frac{6}{5}\,e^{-t}$

b) $y(t) = \frac{1}{2}\text{sen}(t) - \frac{1}{2}\,t\cos(t)$

c) $y(t) = \frac{1}{2}\text{sen}(t) + \frac{1}{2}\,t\cos(t)$

Ej. 12:

a) $y(t) = \frac{1}{6}\,e^{-2t} - \frac{1}{6}\cos(\sqrt{3}\,t)\,e^{t} + \frac{1}{2\sqrt{3}}\,\text{sen}(\sqrt{3}\,t)\,e^{t}$

b) $y(t) = 2 + t - 2\,e^{t} + 2\,t\,e^{t}$

c) $y(t) = \text{sen}(2t)$

d) $y(t) = \frac{1}{2}\,e^{t} + \frac{1}{2}\,\text{sen}(t) + \frac{1}{2}\cos(t)$

e) $y(t) = t^2 - e^{-t}$

f) $y(t) = -2 + 3\,e^{t} - 2\,t\,e^{t}$

Ej. 13:

a) 0 b) $\frac{3}{50}$ c) ¼ d) ln(2) e) -0.03

Ej. 14:

1	2	3	4	5	6	7	8	9	10	11
a	b	c	c	b	c	b	b	c	a	c